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CONVERGENCE OF VORTEX WITH BOUNDARY ELEMENT METHODS*

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Abstract

In this work, the vortex methods for Euler equations with initial boundary value problem is considered, Poisson equations are solved using boundary element methods which can be seen to require less operations to compute the velocity field from the vorticity by Chorin[6]. We prove that the rate of convergence of the boundary element schemes can be independent of the vortex blob parameters.

1. Introduction

The paper written by Chorin[6] in 1973 was the basis of the vortx methods. He divided numerical program into three steps: the first step is to solve the Euler equation with the vortex method, where the velocity flied is computed from the vorticity field with the boundary element method; the second step is to produce the vorticity on the boundary; the third step is to simulate diffusion with random method. It is very difficult that to build the fully mathematic theorey of vortex methods. None can get the convergence of Chorin's algorithm now. In 1978, Chorin, Hughes, McCracken, Marsden[7] regarded the methods as

$$\omega(n \triangle t) = (H(\triangle t)\Theta E(\triangle t))^n \omega_0, \tag{1.1}$$

where Θ is "operator created vorticity", $E(\cdot)$ is Euler's operator, $H(\cdot)$ is Stoke's operator.

People have more studied (1.1) in order to build the mathematical theory of vortex methods. For the simple model, it can be divided as convergence of viscous splitting; convergence of vortex method for Euler equation; and convergence of random vortex method.

The problem in viscous splitting is to consider convergence of the approximate solution, where in every time step, Euler's operator and Stoke's operator both exact, and "operator created vorticity" is considered as a projection operator. Beale and Majda[3] got a fully result for the initial prblems. L. Ying and P. Zhang[21] have studied the initial boundary problems and got a series result. About the random vortex methods, the main result is to see Goodman[8] and D. Long[14]. The convergence of vortex methods for Euler equation is always the main direction. There are many results about the

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convergence of Euler equation for initial problems, we can rerefence Hald[10], Beale and Majda[3,4], Anderson and Greengard[2], Raviart[16] and so. L. Ying[18] first considered the initial boundary problem with extrapolation method and got the convergence of semi-discretization. L. Ying and P. Zhang[20] got completely result for the vortex in finite element method. A similar result was got by P. Zhang[22]. Chorin[6] used the boundary element method to disretize Laplace equation. Since the boundary element method is simple and fast for numerical computation instead of the Green function method, especially for the exterior domain which it is not fit for the finite element method. J. Yang[17] have studied the vortex with boundary element methods. convergence results were given for semi-discretization, but constants in the error bounds depended on the vortex blob parameters.

One purpose of this paper is to prove that the rate of convergence of the boundary element scheme is independent of vortex blob parameters.

2. Boundary Element Method

Boundary element method can be divided into two cases: one is only to consider the error produced by approximation function when the boundary is exact; and the other is both to consider the errors produced by boundary and approximation function. For simplity we only consider the first case.

Let Γ be a smooth curve, $x = x(s)$, $s \in [0, L]$, s is parameter of curve, and $\dfrac{dx}{ds}$ is not zero in any point.

$L = L(\Gamma)$ is the length of curve Γ, if Γ is smooth curves in C^k, then $x(s) \in (C^k)^2$.

We choose NE points A_e $(1 \le e \le NE)$, such that

$$A_e = x(s_e) \qquad 1 \le e \le NE$$

We define $s_0 = 0$, $s_{NE} = L$ and $A_0 = A_{NE}$, $\Gamma = \cup_{e=1}^{NE} \Gamma_e$ for closed curve Γ. Γ_e may be expressed as in the local frame

$$\begin{cases} u = \xi h_e & 0 \le \xi \le 1 \\ v = f_e(\xi) = v_e \circ \overline{A_{e-1}x}(s) \end{cases}$$

where $h_e = |x(s_e) - x(s_{e-1})|$, and denote $h = \max_{1 \le e \le NE}(s_e - s_{e-1})$, and s is function of ξ, their relation is

$$u_e \circ \overline{A_{e-1}x}(s) = \xi h_e,$$

since $x(s)$ is continuous differential, s is uniqne according to ξ if h is small enough.

Denote

$$s = \phi_e(\xi), \ \xi \in [0, 1],$$

ϕ_e is one to one in $[0, 1] \longmapsto [s_{e-1}, s_e]$, while equation of Γ_e in local coordinates (u_e, v_e) is

$$x = \Phi_e(\xi), \qquad \Phi_e(\xi) = x(\phi_e(\xi)) = x(s).$$

If we use $P_m(\xi)$ to express the polynomial function spaces that degree is less than m in $[0, 1]$. Then we can define function spaces P_m^e

$$P_m^e = \{p : p = \widetilde{p} \circ \Phi_e^{-1}, \widetilde{p} \in P_m\},$$