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SOME REMARK ON THE WEIGHTED EULER INTEGRATOR*

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In this short note we extend a result dues to Feng [1] from the classical Hamiltonian systems to the more general Hamilton–Poisson systems and prove that the weighted Euler integrator preserves the Casimir of our Poisson configuration.

Let

$$\dot{x} = \sqcap(x) \cdot \nabla H(x), \; x \in \mathbf{R}^n$$

be a Hamilton–Poisson system, where $\sqcap(x)$ is the matrix of the Poisson structure, H is the Hamiltonian or energy, and let

$$\begin{cases} \Phi_H^h(x^k) = x^{k+1} \\ x^{k+1} = x^k + h \cdot \sqcap(\alpha x^{k+1} + (1-\alpha)x^k) \cdot \nabla H(\alpha x^{k+1} + (1-\alpha)x^k) \end{cases}$$

be the corresponding weighted Euler integrator.

Then a straightforward computation leads us to:

Theorem 1. *If $\sqcap(x) = \sqcap =$ constant then the weighted Euler integrator is a Poisson one, i. e.*

$$D\Phi_H^h(x) \sqcap (D\Phi_H^h(x))^T = \sqcap,$$

if and only if $\alpha = 1/2$.

In the particular case

$$\sqcap(x) = \sqcap = \begin{bmatrix} O_n & I_n \\ -I_n & O_n \end{bmatrix}$$

we recover the result of Feng[1].

Let C be a Casimir of our Poisson configuration $(\mathbf{R}^n, \sqcap(x))$. Then we have:

Theorem 2. *If $\sqcap(x) = \sqcap =$ constant, then the weighted Euler integrator is Casimir preserving.*

Proof. Indeed, we have successively:

$$C(x^{k+1}) - C(x^k) = (\nabla C(x^\star))^T (x^{k+1} - x^k) = h(\nabla C(x^\star))^T \sqcap \nabla H(\alpha x^{k+1} + (1-\alpha)x^k) = 0,$$

as desired.

References

[1] K. Feng, On difference schemes and symplectic geometry, Proceedings of the 1984 Beijing Symposium on Diff. Geometry and Diff. Equations, Computation of Partial Diff. Equations, (ed. K. Feng), Science Press, Beijing, 1985, 42–58.

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