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DISCRETE APPROXIMATIONS FOR SINGULARLY PERTURBED BOUNDARY VALUE PROBLEMS WITH PARABOLIC LAYERS, II*[1)]

P.A. Farrell
(*Department of Mathematics and Computer Science, Kent State University, USA*)

P.W. Hemker
(*CWI, Center for Mathematics and Computer Science, Amsterdam, The Netherlands*)

G.I. Shishkin
(*IMM, Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Science, Ekaterinburg, Russia*)

Abstract

In his series of three papers we study singularly perturbed (SP) boundary value problems for equations of elliptic and parabolic type. For small values of the perturbation parameter parabolic boundary and interior layers appear in these problems. If classical discretisation methods are used, the solution of the finite difference scheme and the approximation of the diffusive flux do not converge uniformly with respect to this parameter. Using the method of special, adapted grids, we can construct difference schemes that allow approximation of the solution and the normalised diffusive flux uniformly with respect to the small parameter.

We also consider singularly perturbed boundary value problems for convection-diffusion equations. Also for these problems we construct special finite difference schemes, the solution of which converges ε-uniformly. We study what problems appear, when classical schemes are used for the approximation of the spatial derivatives. We compare the results with those obtained by the adapted approach. Results of numerical experiments are discussed.

In the three papers we first give an introduction on the general problem, and then we consider respectively (i) Problems for SP parabolic equations, for which the solution and the normalised diffusive fluxes are required; (ii) Problems for SP elliptic equations with boundary conditions of Dirichlet, Neumann and Robin type; (iii) Problems for SP parabolic equation with discontinuous boundary conditions.

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Part II

BOUNDARY VALUE PROBLEM FOR ELLIPTIC EQUATION WITH MIXED BOUNDARY CONDITION

1. Introduction

In this part we sketch a variety of special methods which are used for constructing ε-uniformly convergent schemes. We shall demonstrate a method which achieves improved accuracy for solving singularly perturbed boundary value problem for elliptic equations with parabolic boundary layers.

In Section 4 we shall introduce a natural class, B, of finite difference schemes, in which (by the above mentioned approaches (a) and (b)) we can construct (formally) the special finite difference schemes with approximate solutions which converge parameter-uniformly to the solution of our initial boundary value problem.

In this chapter we consider a class of singularly perturbed boundary value problems which arise when diffusion processes in a moving medium are modeled. For such boundary value problems which describe transfer with diffusion, we construct a special scheme that converges parameter-uniformly. We shall show that for the construction of such schemes from class B, the use of a special condensing grid (or an adaptive mesh) is necessary. It means that the choice (to construct special parameter-uniformly convergent schemes for our class of convection diffusion problems) is quite restricted. By condensing (or adaptive) grids we can construct finite difference schemes which converge parameter-uniformly. We shall present and discuss the results of numerical computations using both the classical and the new special finite difference schemes.

2. The Class of Boundary Value Problems

2.1. The physical problem

The diffusion of a substance in a convective flow of an incompressible fluid in a two-dimension domain gives rise to an equation of the form

$$-\varepsilon\Delta u(x) + \vec{v}(x)\cdot\vec{\nabla}\, u(x) = F(x),\ \ x \in \Omega, \tag{2.1a}$$

where $\vec{v}(x)$ and $F(x)$ are the velocity and source, respectively; $1/\varepsilon$ is the Peclet number (Reynolds number), if the substance is heat (diffusive matter or momentum)[?]. When the substance is heat (diffusive matter or momentum) then $u(x)$ is the temperature (density or velocity) at the point x. On the boundary of domain considered (that is the wall of the container holding the fluid) we have a boundary condition that describes the exchange of the substance with the surrounding environment

$$-\alpha(u(x) - U(x)) - \frac{\partial}{\partial n}u(x) = 0,\ \ x \in \partial^0\Omega. \tag{2.1b}$$