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THE GLOBAL CONVERGENCE OF THE GMED ITERATIVE ALGORITHM*

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Abstract

In the late 1970's, Wiggins proposed a minimum entropy deconvolution (MED) which has become one of the most important deconvolution methods. He gave a varimax norm V_2^4 and a MED iterative procedure. Fortunately, for the last ten years in the practical using, the MED algorithm has never failed to reach a maximizer of the varimax norm. But so far, no theoratical proof has been given to show the convergence of the MED procedure. In this paper, we prove the global convergence of a generalized MED iterative procedure with respect to a generalized varimax norm $V_q^p(q = 2, p > 2)$.

§1 Introduction

The minimum entropy deconvolution (MED) was proposed by Wiggins in the late 1970's and is based on the assumption of spiky and sparse appearance of the reflectivity series, instead of the minimum phase source signals and the white reflectivity series. Because of its new idea, simple iterative algorithm and weak hypotheses over the components, etc, it has attracted attention and become one of the most important deconvolution methods [2,3]. The detailed studies about the mathematical theories and multi-maximizer property of the MED have been given in [4,5,6,7]. Many other varimax norms and generalized MED methods suitable for various practical applications have been proposed in [8,9,10].

The generalized minimum entropy deconvolution (GMED) norm is

$$V_q^p = \frac{\sum_{i=1}^{n} |y_i|^p}{\left(\sum_{i=1}^{n} |y_i|^q\right)^{\frac{p}{q}}} = \max, \quad p, q > 0;\ p > q, \tag{1.1}$$

where $\{y_i\}_{i=1}^n$ is the output of the filtering. when $p = 4, q = 2$, the GMED norm is just the wiggins' MED. A MED iterative algorithm for solving the preceding problem was given and experiments evidenced that it worked well. Many tests were made in [7] with different initial points, and none of them failed to reach a maximizer of V_2^4 . But no one could be sure that there were no exceptions. Hence, a theoretical verification of the algorithm's convergence is necessary.

In this paper, we generalize the iterative procedure of V_2^4 by that of $V_q^p(q = 2,\ p > q)$, and a proof of its global convergence is given.

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§2 The Iterative Procedure (GMED) of the Varimax Norm V_2^p

In this paper, the underlying field of a vector space is a set $\Re$ of the real numbers. $\Re^{m\times n}$ denotes the set of $m\times n$ matrixes; $\Re^m$, the set of the m-dimensional vectors. If not otherwise specified, a capital letter stands for a vector, and the corresponding common letter with a subscript represents a component of the vector.

Set $A \in \Re^{m\times n}$. Ker A is the null of A, and Im A is the image of A. Namely,

$$\mathrm{Ker}A \equiv \{U \in \Re^n \mid AU = 0\},$$

$$\mathrm{Im}A \equiv \{V \in \Re^m \mid V = AU, \text{ for any } U \in \Re^n\}.$$

Suppose $Y \in \Re^n, \alpha > 0$. Then, $Y^\alpha \in \Re^n$, $| Y | \in \Re^n$. That is,

$$Y^\alpha = (y_1 \mid y_1^{\alpha-1} \mid, y_2 \mid y_2^{\alpha-1} \mid, ..., y_n \mid y_n^{\alpha-1} \mid)^\tau,$$

$$| Y | = (| y_1 |, | y_2 |, ..., | y_n |)^\tau.$$

The minimum entropy deconvolution filter is a linear filter, where the observed signals are

$$W = (x_1, x_2, ..., x_m)^\tau.$$

Without loss of generality, let $x_1 \neq 0$. If the filter $F \in \Re^l$ $(F \neq 0)$, then its output $Y \in \Re^n$, $n = m + l - 1$, and by the convolutional property of the filter, we have

$$Y = XF. \tag{2.1}$$

That is, $Y \in \mathrm{Im}X$, where

$$X = \begin{pmatrix} x_1 & & & \\ x_2 & x_1 & & \\ \vdots & x_2 & \ddots & x_1 \\ x_m & \vdots & \ddots & x_2 \\ & x_m & & \vdots \\ & & & x_m \end{pmatrix}_{n\times l}. \tag{2.2}$$

Obviously, rank $X = l$, Ker $X = \{0\}$, and $X^\tau X > 0$ [11].

By convention, we bave

$$\|Y\|_p = \Big(\sum_{i=1}^n |y_i|^p\Big)^{\frac{1}{p}}, \quad p \geq 1 \quad \text{(the Hölder norm)};$$

$$\|Y\|_\infty = \max_i |y_i| \quad \text{(the infinity norm)}.$$

Definition 2.1. *The generalized minimum entropy deconvolution of varimax norm V_q^p is the following maximizing problom:*

$$\mathrm{GMED}: \quad \max_{Y=XF} V_q^p = \max_{Y=XF} \Big(\frac{\| Y \|_p}{\| Y \|_q}\Big)^p, \quad p > q. \tag{2.3}$$