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ON THE RELATION BETWEEN AN INVERSE PROBLEM FOR A SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS AND AN INITIALBOUNDARY VALUE PROBLEM FOR A HYPERBOLIC SYSTEM *1)

Shao Xiu-min Yuan Zhong-min
(*Institute of Mathematics, Academia Sinica, Beijing, China*)

Abstract

This paper deals with a coefficient inverse problem of a system of ODEs whose coefficient matrix is the so-called generalized negative definite matrix. To solve the problem, an initial-boundary value problem of a hyperbolic system of PDEs is constructed. The existence and uniqueness of its solution and its asymptotic convergence with respect to one of the variables to the original inverse problem are proved. As a result, the solution of the inverse problem is reduced to the solution of the direct problem. A few numerical examples were solved to show the effectiveness of the method.

§1. Introduction

In this paper, the following coefficient inverse problem of a system of ODEs is discussed:

$$\frac{dx(t)}{dt} = Ax(t) + F(t) \tag{1.1}$$

where $x(t)$ and $F(t)$ are given, and A is a generalized negative definite $n \times n$ unknown matrix. Here by generalized negative definite matrix we mean the real parts of eigenvalues of which are all negative and there exists a constant $C > 0$ such that the inequality

$$(Ax, x) \le -C(x, x) \tag{1.2}$$

is valid for all $x \in R^n$.

Such kind of problems is often met in practice. For example, coefficient inverse problems of parabolic differential equations can be reduced to it, if the spatial variables are discretized or integral transformations with respect to the spatial variables are applied. In this case the matrix A is a strict negative definite matrix. Another important example is the identification problem of compartmental models, which have been widely used in a variety of areas, such as medicine, biology, economics, and so on. In this problem the matrix A is the compartmental matrix with the properties: 1) Its off-diagonal entries are nonnegative; 2) Its diagonal entries are negative; 3) It is diagonally dominant with respect to the columns.

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It is well known that the real parts of eigenvalues of the matrices which possess the Properties 2) and 3) are negative. Under some additional conditions the compartmental matrices satisfy inequality (1.2) so that they are generalized negative definite.

In 1983, H.W. Alt et al.[1] developed an elegant method for solving coefficient inverse problems of differential equations. They considered the elliptic equation

$$-\nabla\cdot(a\nabla u)=f, \tag{1.3}$$

where the solution $u(x,y)$ is given, and the problem is to find a positive definite matrix $a(x,y)$ to fit (1.3). They have proved that the solution of the inverse problem mentioned above is the limit of the solution of a certain direct problem of a system of partial differential equations when one of its parameters tends to infinity (in the rest part of this paper we will call this direct problem asymptotically convergent to the original inverse problem). In this way the solution of the inverse problem is reduced to the solution of the direct problem, for which there are well known methods, thus making the things much easier.

In this paper, a similar approach is used for solving (1.1). Differently from [1], a system of evolutionary equations is considered here. Instead of discretizing the time variable in advance (as the authors of [1] suggested), we consider an auxiliary direct problem of a hyperbolic system of differential equations which asymptotically converges to the inverse problem (1.1) itself, rather than to its time discrete form. Moreover, in [1], all results are obtained only for the finite dimensional approximation of the original problem. The striking difference of this paper is that we have proved the asymptotic convergence for the infinite dimensional case.

§2. An Auxiliary Problem

We write the problem which was proposed at the beginning of this paper as follows.

Problem (P): Suppose that $F^*(t)$ and $x^*(t)$ are given. Find a generalized negative definite matrix A^* such that

$$\frac{dx^*(t)}{dt}=A^*x^*(t)+F^*(t). \tag{2.1}$$

Here the existence of the solution A^* is assumed.

To solve this inverse problem, we introduce an auxiliary direct problem

$$\left.\begin{aligned}&\frac{\partial x(t,\tau)}{\partial\tau}+\frac{\partial x(t,\tau)}{\partial t}-A(\tau)x(t,\tau)-F^*(t)=0 &(2.2)\\ &\frac{dA(\tau)}{d\tau}+\frac{1}{T}\int_0^T[x(t,\tau)-x^*(t)]\otimes x(t,\tau)dt=0 &(2.3)\end{aligned}\right\},\quad (t,\tau)\in(0,T)\times(0,\infty),$$

$$x(t,0)=x_0(t),\quad t\in[0,T], \tag{2.4}$$

$$x(0,\tau)=x^*(0),\tau\in[0,\infty), \tag{2.5}$$

$$A(0)=A_0, \tag{2.6}$$

where the operation $\otimes$ is defined for two R^n-vectors V_1,V_2 by

$$V_1\otimes V_2:=(v_{1i}v_{2j}),\quad i,j=1,2,\cdots,n\in R^{n\times n},$$

$x_0(t)$ is an arbitrary vector function, and A_0 arbitrary matrix.