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G^1 SMOOTHING SOLID OBJECTS BY BICUBIC BEZIER PATCHES*

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Abstract

A general and unified method is presented for generating a wide range of 3-D objects by smoothing the vertices and edges of a given polyhedron with arbitrary topology using bicubic Bezier patches. The common solution to the compatibility equations of G^1 geometric continuity between two Bezier patches is obtained and employed as the foundation of this new method such that this new solid and surface model is reliable and compatible with the solid modeling and surface modeling systems in the most common use. The new method has been embeded in an algorithm supported by our newly developed solid modeling system MESSAGE. The performance and implementation of this new algorithm show that it is efficient, flexible and easy to manipulate.

§1. Introduction

In recent years, much effort has been put to developing more reliable and flexible solid modeling systems and surface modeling systems to meet the needs in industry. Combining the surface modeling and solid modeling is a new trend in computer graphics, CAD/ CAM and their applications. The application of surface modeling techniques within a solid modeling system requires a general and unified method to generate a wide range of 3-D objects bounded by planar and Bezier patches. Especially, in shape design, it is very common and important to creat a wide range of objects from polyhedra to free form shape in one system.

Quite a number of solid and surface modeling systems have adopted more flexible mathematical models such as B-reps, CSG, Bezier patches, B-spline surfaces, Coons patches and so on. Recently, the topic of integration of surface modeling with solid modeling has received much attention [1]-[2]. For the description of many objects, both the flexibility of the shape controlling of free-form surfaces and compatible representation of solid modeling techniques must be provided. Unfortunately, how to generate a wide range of 3-D objects from a polyhedron such that the model is consistent with the most solid modeling system is still a problem.

Some useful methods for smoothing vertices and edges of polyhedra have been proposed by Doo et al.[3]-[4], Lu et al.[5], H.Chiyokura and F.Kimura et al.[6]-[7] and J.R. Rossignac and A.A.G. Requicha et al.[8]-[9]. In [3], [4], extraordinary points are represented by a lot of subdivided patches, and globally rounded surfaces are generated from polyhedra. But it is difficult to round off a solid locally or generate sharp edge curves, and it is difficult to do any analysis because it is not described in mathematical expression and analytic form. In [6] a method is proposed for rounding off corners and edges of polyhedra using Gregory patches

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and applied to MODIF system successfully. The Gregory patches are neither represented by Bezier nor B-spline surfaces and are inconsistent with most solid and surface modeling systems. In [8], [9] some blending methods were proposed, but they cannot be used for rounding off corners. The main difficulties in solving this problem come from the smooth joint between free-form surfaces. Unfortunately, the conditions of geometric continuity and its solutions are still a considerable problem.

In this paper the common solutions to the compatible equations of geometric continuity of first order (denoted by G^1) between two Bezier patches are obtained from which we drive the condition of G^1 geometric continuity (Fig. 1)

$$r(1,v) = \bar{r}(\bar{u},0), \quad 0 \le \bar{u} = v \le 1.$$

It is readily shown that if S and $\bar{S}$ meet with G^1 along Γ, then there exist the following rational polynomials

$$p(v) = \sum_{i=0}^{5} e_i v^i \Big/ \sum_{i=0}^{5} d_i v^i, \tag{7}$$

$$q(v) = \sum_{i=0}^{6} f_i v^i \Big/ \sum_{i=0}^{5} d_i v^i, \tag{8}$$

such that

$$\left.\frac{\partial \bar{r}}{\partial \bar{u}}\right|_{\bar{v}=1} = p(v)\left.\frac{\partial r}{\partial u}\right|_{u=0} + q(v)\left.\frac{\partial r}{\partial v}\right|_{u=0}, \quad 0 \le v = \bar{u} \le 1. \tag{9}$$

Substituting (7) and (8) into (9) and comparing the coefficients on both sides yield

$$M_d^1(\bar{Q}_{10}, \bar{Q}_{11}, \bar{Q}_{12}, \bar{Q}_{13})^T = M_e^1(\bar{P}_{20}, \bar{P}_{21}, \bar{P}_{22}, \bar{P}_{23})^T + M_f^1(\bar{P}_{31}, \bar{P}_{32}, \bar{P}_{33})^T, \tag{10}$$

$$M_d^2(\bar{Q}_{10}, \bar{Q}_{11}, \bar{Q}_{12}, \bar{Q}_{13})^T = M_e^2(\bar{P}_{20}, \bar{P}_{21}, \bar{P}_{22}, \bar{P}_{23})^T + M_f^2(\bar{P}_{31}, \bar{P}_{32}, \bar{P}_{33})^T, \tag{11}$$

where

$$M_d^1 = \begin{bmatrix} d_0 & & & \\ d_1 & d_0 & & \\ d_2 & d_1 & d_0 & \\ d_3 & d_2 & d_1 & d_0 \end{bmatrix}, \quad M_e^1 = \begin{bmatrix} e_0 & & & \\ e_1 & e_0 & & \\ e_2 & e_1 & e_0 & \\ e_3 & e_2 & e_1 & e_0 \end{bmatrix}, \quad M_f^1 = \begin{bmatrix} f_0 & & \\ 2f_1 & 3f_0 & \\ f_2 & 2f_1 & 3f_0 \\ f_3 & 2f_2 & 3f_1 \end{bmatrix},$$

$$M_d^2 = \begin{bmatrix} d_4 & d_3 & d_2 & d_1 \\ d_5 & d_4 & d_3 & d_2 \\ & d_5 & d_4 & d_3 \\ & & d_5 & d_4 \\ & & & d_5 \end{bmatrix}, \quad M_e^2 = \begin{bmatrix} e_4 & e_3 & e_2 & e_1 \\ e_5 & e_4 & e_3 & e_2 \\ & e_5 & e_4 & e_3 \\ & & e_5 & e_4 \\ & & & e_5 \end{bmatrix}, \quad M_f^2 = \begin{bmatrix} f_4 & 2f_3 & 3f_2 \\ f_5 & 2f_4 & 3f_3 \\ f_6 & 2f_5 & 3f_4 \\ & 2f_6 & 3f_5 \\ & & 3f_6 \end{bmatrix}.$$

From (10) we obtain $(\bar{Q}_{10}, \bar{Q}_{11}, \bar{Q}_{12}, \bar{Q}_{13})$. Substituting it into (11) we have some conditions on the coefficients d_i, e_i and f_i which are called shape parameters. Notice that the shape parameters must be independent of the control points. Thus we have

$$M_e^2 = M_d^2 (M_d^1)^{-1} M_e^1, \tag{12}$$

$$M_f^2 = M_d^2 (M_d^1)^{-1} M_f^1. \tag{13}$$