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# A MODIFIED MACCORMACK'S SCHEME*

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**Abstract**

In this paper a modified MacCormack's scheme is presented. The scheme is based on flux vector splitting. The test computations show that the proposed modified scheme produces much better numerical results than original MacCormack's scheme.

## §1. Introduction

A numerical flux function plays a very important role in solving hyperbolic equations of conservation laws by finite difference methods. The first order accurate numerical flux scheme may be most dependable in providing solutions which are free of computational noise, but it possesses sufficiently large dissipative trunction error so that discontinuities are smeared out on grids.

The higher order numerical flux schemes[5,7] possess the peculiar property that nonphysical oscillations in the solution can be generated in the vicinity of steep gradient regions[12]. This computational noise may degrate or destroy the accuracy of solution. Undesirable physical features of the simulated flow such as negative masses or energy densities may develop in solving gasdynamic equations.

As a consequence, the development of higher order monotonic or TVD numerical schemes has continued. Some examples of these schemes are in [1,2,10]. In these schemes nonlinear filtering techniques are used for higher order numerical algorithms. In general, constraints are imposed on the gradients of the dependent variable [9] or on the gradients of the flux functions [4] in these algorithms. This technique effectively removes computation noise from steep gradient solutions.

A modified MocCormack's scheme is presented in this paper. The algorithm is based on flux vector splitting [8] for systems and the flux limeter may be treated in a relatively simple and convenient way. The test calculations show that the numerical results are of higher resolution.

## §2. Description of The Algorithm

In this section we will briefly describe the general algorithm. A modified MacCormack's scheme is described in the next section.

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To illustrate the basic notion we consider the numerical algorithm for a one-dimensional system of conservation laws

$$\frac{\partial W}{\partial t} + \frac{\partial F(W)}{\partial x} = 0, \tag{1}$$

where $W$ and the flux function $F(W)$ are $m$-component column vectors.

Now computing $x$-spatial is divided into cells of equal width, $\Delta x$. We take $W$ at the center of the $j$th cell. At time $n\Delta t$, these variables, $W_j^n$, are known on the cells. The objective is to compute the dependent variables one time step later, $W_j^{n+1}$. The algorithm is accomplished as follows

**Step 1.** The predictor solution is calculated at $(n+1)\Delta t$ using a first order accurate scheme for computing the spatial derivative in (1)

$$\widetilde{W}_j = W_j^n - \lambda\left(F_{j+\frac{1}{2}}^{nf} - F_{j-\frac{1}{2}}^{nf}\right), \tag{2}$$

where $\lambda = \dfrac{\Delta t}{\Delta x}$ and $F_{j+\frac{1}{2}}^{nf}$ is a first order numerical flux at the boundary between the $j$th and $(j+1)$th cells. An example of a first order scheme is the upwind method [3]. This step should not introduce ripples into the solution.

**Step 2.** An anti-diffusion flux is calculated

$$F_{j+\frac{1}{2}}^{a} = F_{j+\frac{1}{2}}^{nh} - F_{j+\frac{1}{2}}^{nf}, \tag{3}$$

where $F_{j+\frac{1}{2}}^{nh}$ is a higher order approximation to the spatial derivative in (1). The solutions computed by the higher order flux, $F_{j+\frac{1}{2}}^{nh}$, contain ripples.

**Step 3.** The predictor solution which is monotonic, $\tilde{W}$, and the higher order flux are used in conjunction with a nonlinear filter to obtain the resulting solutions at time $(n+1)\Delta t$

$$W_j^{n+1} = \tilde{W}_j - \lambda(F_{j+\frac{1}{2}}^{ac} - F_{j-\frac{1}{2}}^{ac}). \tag{4}$$

The objective is to control the flux , $F_{j+\frac{1}{2}}^{ac}$, in and out of cells so as to decrease the diffusion introduced by the first order scheme. Here $F_{j+\frac{1}{2}}^{ac}$ is adjusted as follows

$$F_{j+\frac{1}{2}}^{ac} = \begin{cases} S_{j+\frac{1}{2}} \min(\alpha|F_{j+\frac{3}{2}}^{a}|, |F_{j+\frac{1}{2}}^{a}|, \alpha|F_{j-\frac{1}{2}}^{a}|), & \text{when} \\ \qquad S_{j+\frac{1}{2}} = \mathrm{Sign}(F_{j+\frac{1}{2}}^{a}) = \mathrm{Sign}(F_{j+\frac{3}{2}}^{a}) = \mathrm{Sign}(F_{j-\frac{1}{2}}^{a}); \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

where $\alpha$ is a constant between 1 and 2. From (4) and (2) we can see that the scheme is of higher order if $F_{j+\frac{1}{2}}^{ac} = F_{j+\frac{1}{2}}^{a}$, and it is the first order scheme (2) when parameter $\alpha = 0$.

Using the algorithm described above we have constructed some schemes already. One of them is a modified MacCormack's scheme in the next section.

## §3. Modified MacCormack's scheme

As well known, MacCormack's second order scheme[7] for the one- dimensional system of conservation laws (1) is

$$\tilde{W}_j = W_j^n - \lambda(F_j^n - F_{j-1}^n), \tag{6.1}$$