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BAROCLINIC MATHEMATICAL MODELING OF FRESH WATER PLUMES IN THE INTERACTION RIVER-SEA

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Abstract. The estuarine zone is an area of strong interaction between fresh and salty water. Dynamics in these areas is complex due to the interaction of the forcing mechanisms such as wind, tides, local coastal currents and river discharges. The difference of density between fresh water and salted water causes the formation of the buoyant plumes which have been investigated by means of numerical models and field studies. Plumes play a significant role in the transport of pollutants and the ecology in the frontal areas where density gradients are strong. Therefore, in order to investigate the horizontal and vertical dispersion of salinity and temperature the YAXUM/3D baroclinic numerical model was developed. The model is validated and applied for two particular cases. The first one consist of modeling the discharge of a jet of hot water where the gradients of temperature prevail and the second to study the discharge of the mouth of the estuary Leschenault toward the Koombana bay, Australia where salinity gradients are analyzed. The results derived from the YAXUM/3D are satisfactory and in agreement to with other models which have been already validated.

Key Words. estuarine zone, baroclinic modeling, buoyant plume, vertical mixing.

1. Introduction

The estuarine zone is a complex area due to the interaction of wind, tides, local coastal currents and river discharges. In this area, the fresh water moves toward the sea on top of the salty water layer. The dynamics of the frontal area play an important role in the biology of the area due to the accumulation of particulate organic matter. In addition, the daily heating and cooling effect produce changes of temperature in both rivers and marine waters [1].

In coastal areas, the interaction of fresh water river discharges into the sea causes the formation of the buoyant plumes. In order to investigate de dynamics of buoyant plumes laboratory, field measurements and numerical simulations have been carried out thoroughly. Also, a significant ecological impact has been observed due to amount of particles and pollutants brought along with the river flow.

Numerical models have proven to be a successful tool to investigate buoyant plumes. So different environmental conditions can be simulated in relatively short periods of time. Oceanographers have established different approaches to classify their own models. One of the most important approaches in the literature is the

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consideration of the density variation, where the models are classified as barotropic or baroclinics. Although this approach is derived from the oceanic classification models, it is wise to take it into account in what refers to applications in estuaries, outlets or coastal lagoons, because in some cases, important processes exist due to the change of densities.

The difference between a barotropic and baroclinic models resides in the vertical discretization and on the determination of the pressure term in the Reynolds equations. In the barotropic models the vertical integration is applied and therefore, density is uniform with depth, while in the baroclinic models vertical process are considered such as gradients of temperature, salinity and density.

In this paper, a baroclinic numerical model is developed and validated for buoyant fresh water plumes discharging to the coastal environment. In the first part, the numerical model is described and in the second part the validation and applications examples are showed. The results are compared to those derived by McGuirk and Rodi [2]. In the second application a dispersion of fresh water plume into a marine environment is modeled. Basically, attention is paid to the simulation of the salinity behavior. In this case we are reproducing the work developed by Okely [3], who works with another numerical model whose application was given for the Koombana Bay in Australia.

2. The Numerical Model

The YAXUM/3D numerical model was developed and solves the three dimensional equations for a free surface flows based on the numerical scheme proposed by Casulli and Cheng[4], where the numerical solution is given by a combination of a semi-implicit Eulerian-Lagrangian numerical scheme.

2.0.1. Governing equations. These equations describe the velocity fields and the free surface variations. The density is solved by means of a state equation in function of a temperature, salinity and pressure fields. The model solves a salinity and temperature transport equations. For the pressure two kinds of approximations are taken in account. The first one is the hydrostatic approach where the pressure P changes with the depth (z), according to

$$\frac{\partial P}{\partial z} = -\rho g \tag{1}$$

This relation is valid if the horizontal dimension is larger than the vertical one, which is the main consideration for the shallow water equations approach.

The second consideration is named the Boussinesq approximation, where density may be considered as a constant in all terms, except the gravitational term.

Horizontal velocities:

$$\frac{\partial \overline{U}}{\partial t} + \overline{U}\frac{\partial \overline{U}}{\partial x} + \overline{V}\frac{\partial \overline{U}}{\partial y} + \overline{W}\frac{\partial \overline{U}}{\partial z} = -\frac{1}{\rho_0}\frac{\partial \overline{P}}{\partial x} - \frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y} - \frac{\partial \overline{u'w'}}{\partial z} + f\overline{V} \tag{2}$$

$$\frac{\partial \overline{V}}{\partial t} + \overline{U}\frac{\partial \overline{V}}{\partial x} + \overline{V}\frac{\partial \overline{V}}{\partial y} + \overline{W}\frac{\partial \overline{V}}{\partial z} = -\frac{1}{\rho_0}\frac{\partial \overline{P}}{\partial y} - \frac{\partial \overline{v'u'}}{\partial x} - \frac{\partial \overline{v'v'}}{\partial y} - \frac{\partial \overline{v'w'}}{\partial z} - f\overline{U} \tag{3}$$

Vertical velocity:

$$\frac{\partial \overline{W}}{\partial z} = -\left(\frac{\partial \overline{U}}{\partial x} + \frac{\partial \overline{V}}{\partial y}\right) \tag{4}$$