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AN ITERATIVE ALGORITHM FOR THE COEFFICIENT INVERSE PROBLEM OF DIFFERENTIAL EQUATIONS*

Chao Hong-yang
(*Zhongshan University, Guangzhou, China*)

Abstract

In this paper, an iterative algorithm for solving a coefficient inverse problem is submitted. The key of the method is to project an unknown coefficient function on a finite dimensional function space. Thus, the inverse problem can be changed into a nonlinear algebraic system of equations.

§1. Introduction

Now, the coefficient inverse problem of differential equations is becoming more and more noticeable in the fields of economics, science and technology, national defence and so on. It can be applied to many aspects such as resources prospecting, system identification, telemetering and remote sensing, etc. Many authors studied the problem and presented a lot of valuable results in theoretical analysis and numerical computation[1,2,3]. However, since the inverse problem is nonlinear and ill-posed, much difficulty remains in theoretical analysis and numerical solution. So, there are still a lot of problems to be solved in both respects.

We will take the coefficient inverse problem of the one-dimensional convection-diffusion equation for example and derive an algorithm to solve it.

Consider the initial-boundary problem

$$\frac{\partial u}{\partial t} - \frac{\partial}{\partial x}(k(x)\frac{\partial u}{\partial x}) + v(x,t)\frac{\partial u}{\partial x} = f(x,t), \quad 0 < x < 1, \quad t > 0, \tag{1.1}$$

$$u(x,0) = u_0(x), \quad 0 < x < 1, \tag{1.2}$$

$$u(0,t) = g_0(t), \quad u(1,t) = g_1(t), \quad t > 0, \tag{1.3}$$

with additional measured condition

$$Pu(\cdot,t) = h(t), \quad t > 0, \tag{1.4}$$

where $k \in æ = \{k \in H^1[0,1]; k(x) \geq k_* > 0, x \in (0,1)\}, v, f, g_0, g_1$ and h are given functions, and P is an operator, for example

$$Pu(\cdot,t) = u(x_0,t), \quad x_0 \in (0,1) \tag{1.5}$$

or

$$Pu(\cdot,t) = \frac{\partial u}{\partial x}(0,t), \tag{1.6}$$

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and so on.

By the general theory of differential equations, the (classical or generalized) solution of problem (1.1)–(1.3) exists and is unique under certain conditions. But now, we aim at finding $k \in æ$, such that the solution $u = u(k; x, t)$ of problem (1.1)–(1.3) satisfies the additional measured condition (1.4). This is the "inverse problem".

This problem has an important background in physics. If $v = 0$, (1.1) is a heat conduction equation, the problem is to find the thermal conductivity of an inhomogeneous material[5]. If $v \neq 0$, it is a problem of identification of the diffusion coefficient in a water quality model for a river[4]. The existence and uniqueness of the solution of the inverse problem have been discussed in [6]; we only put forward an algorithm to solve it.

§2. An Iterative Algorithm

Through a simple map, (1.1)–(1.4) can be changed into the following form

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial}{\partial x}(k(x)\dfrac{\partial u}{\partial x}) + v(x,t)\dfrac{\partial u}{\partial x} = f(x,t) - g(t)k'(x), \quad 0 < x < 1, \quad t > 0, & (2.1) \\ u(x,0) = u_0(x), \quad 0 < x < 1, & (2.2) \\ u(0,t) = u(1,t) = 0, \quad t > 0 & (2.3) \end{cases}$$

and

$$Pu(\cdot, t) = h(t), \tag{2.4}$$

where v, f, g, u_0, and h are given functions.

2.1. Approximation of the inverse problem

Suppose $e_1, \cdots, e_n$ are linearly independent functions in $H_0^1[0,1]$, and $E_n =$ span $\{e_1, \cdots, e_n\}$. For any $k \in æ$, let

$$u^n = \sum_{j=1}^{n} c_j(t)e_j(x)$$

be an approximate solution of problem (2.1)–(2.3).

Taking the inner product of $e_j (j = 1, \cdots, n)$ with equation (2.1) in $L^2[0,1]$, we have

$$(\frac{\partial u}{\partial t}, e_j) + (k\frac{\partial u}{\partial x}, e_j') + (v\frac{\partial u}{\partial x}, e_j) = (f, e_j) + g(t)(k, e_j'), \quad j = 1, \cdots, n.$$

Substituting u by $u^n = \sum_{j=1}^{n} e_j(x)c_j(t)$ in the above formula and making a simple arrangement, we obtain

$$A_n\frac{dC}{dt} + [B_n(k) + V_n(t)]C(t) = F_n(t) + g(t)R_n(k),$$