A SPECTRAL METHOD FOR A CLASS OF SYSTEM OF MULTI-DIMENSIONAL NONLINEAR WAVE EQUATIONS *

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In [1,2], the problem of three-dimensional soliton of a class of system for three-dimensional nonlinear wave equations was investigated, and the existence and stability of three-dimensional soliton was proved. In [3] the system discussed in [1,2] was generalized and a more general class of system of multi-dimensional nonlinear wave equations were studied. It was proved that the solution of its initial-boundary value problem was well posed under some conditions. This system has been studied by the finite difference method and the finite element method [4,5]. In this paper, we take the trigonometric functions as a basis to derive a spectral method for the system and give a strict error analysis in theory.

1. Notations and Statement of the Problem

We consider the periodic initial value problem of a system of nonlinear wave equations

$$\Box\varphi+\mu^2\varphi+\nu^2\chi^2\varphi+f(|\varphi|^2)\varphi=0,\quad (x,t)\in\Omega\times(0,T], \tag{1.1}$$

$$\Box\chi+\delta^2\chi+\nu^2\chi|\varphi|^2+h(\chi)=0,\quad (x,t)\in\Omega\times(0,T], \tag{1.2}$$

with initial conditions

$$\varphi|_{t=0}=\varphi_0(x),\ \frac{\partial\varphi}{\partial t}|_{t=0}=\varphi_1(x),\quad \chi|_{t=0}=\chi_0(x),\ \frac{\partial\chi}{\partial t}|_{t=0}=\chi_1(x),\ x\in\Omega, \tag{1.3}$$

where $\Omega=[-\pi,\pi]^n$, $\Box=\dfrac{\partial^2}{\partial t^2}-\sum\limits_{i=1}^{n}\dfrac{\partial^2}{\partial x_i^2}$, and $\varphi(x,t),\chi(x,t)$ are unknown complex and real periodic functions respectively. $\varphi_0(x),\varphi_1(x),\chi_0(x),\chi_1(x)$ are known complex and real valued periodic functions respectively. All of them have the period 2π for $x_s, 1\le s\le n, n\le 3$. $f(s)$ and $h(s)$ are known real functions. μ,ν,δ are real constants.

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Let I denote the integers. $l=(l_1,\cdots,l_n)\in I^n, |\, l\,| = \max\limits_{1\le i\le n}|l_i|$ and

$$S_N=\text{Span}\Big\{\psi_l=e^{il\cdot x}|l\in I^n, |\,l\,|\le N\Big\}.$$

Put $u(x,t)=\sum\limits_{l\in I^n}u_l(t)e^{il\cdot x}$ and $u^{(N)}(x,t)=\sum\limits_{|l|\le N}u_l(t)e^{ix\cdot l}, R^{(n)}(u(x,t))=u(x,t)-u^{(N)}(x,t)$. Define $(u,v)=\int_\Omega u\bar{v}dx$. Let $H_p^s(\Omega)$ denote the Sth–order Sobolev spaces of real or complex valued periodic functions with the norm $\|\cdot\|_s$. For $r\ge 1$, we denote $\|u\|_{L^r}=(\int_\Omega |u|^r dx)^{1/r}$ and $\|u\|_{L^2}=\|u\|$.

Let τ be the mesh spacing in time and

$$u_t(x,t)=\frac{1}{\tau}(u(x,t+\tau)-u(x,t)),\quad u_{\bar t}(x,t)=\frac{1}{\tau}(u(x,t)-u(x,t-\tau)).$$

In this paper, we assume

(1) $f,h\in C^2$, and $|f(s)|\le A_0 s, |f'(s)|\le A_1, s\ge 0; |h(s)|\le B_0|s|^3, |h'(s)| \le B_1 s^2$, where $A_i, B_i,\ \ i=0,1$, are positive constants;

(2) $F(s)=\int_0^s f(z)dz,\ \ H(s)=\int_0^s h(z)dz,\ \ F(s)\ge 0, s\ge 0,\ \ H(s)\ge 0, s \in(-\infty,+\infty)$;

(3) system (1.1)–(1.3) has solutions, which and the initial data are properly smooth. For (1.1)–(1.3) we construct the following conservative fully discrete spectral scheme

$$\begin{aligned}&(\Phi_{t\bar t}^{(N)}(t),\psi_j)+\tfrac{1}{2}(\nabla(\Phi^{(N)}(t+\tau)+\Phi^{(N)}(t-\tau)),\nabla\psi_j)+\tfrac{\mu^2}{2}(\Phi^{(N)}(t+\tau)\\ &+\Phi^{(N)}(t-\tau),\psi_j)+\tfrac{\nu^2}{2}((\Sigma^{(N)}(t))^2(\Phi^{(N)}(t+\tau)+\Phi^{(N)}(t-\tau)),\psi_j)\\ &+\tfrac{1}{2}(F(|\Phi^{(N)}(t+\tau)|^2,|\Phi^{(N)}(t-\tau)|^2)(\Phi^{(N)}(t+\tau)+\Phi^{(N)}(t-\tau)),\psi_j)=0,\end{aligned}\tag{1.4}$$

$$(\Phi^{(N)}(0),\psi_j)=(\varphi_0(x),\psi_j),\tag{1.5}$$

$$\begin{aligned}&(\Sigma_{t\bar t}^{(N)}(t),\psi_j)+\tfrac{1}{2}(\nabla(\Sigma^{(N)}(t+\tau)+\Sigma^{(N)}(t-\tau)),\nabla\psi_j)+\tfrac{\delta^2}{2}(\Sigma^{(N)}(t+\tau)\\ &+\Sigma^{(N)}(t-\tau),\psi_j)+\tfrac{\nu^2}{2}(|\Phi^{(N)}(t)|^2(\Sigma^{(N)}(t+\tau)+\Sigma^{(N)}(t-\tau)),\psi_j)\\ &+(H(\Sigma^{(N)}(t+\tau),\Sigma^{(N)}(t-\tau)),\psi_j)=0,\end{aligned}\tag{1.6}$$

$$\Big(\Sigma^{(N)}(0),\psi_j\Big)=(\chi_0(x),\psi_j),\quad |j|\le N,\tag{1.7}$$

where

$$F(z_1,z_2)=\begin{cases}\dfrac{1}{z_2-z_1}\displaystyle\int_{z_1}^{z_2}f(s)ds, & \text{if } z_1\ne z_2,\\ f(z_1), & \text{if } z_1=z_2,\end{cases}$$