Vol.7 No.2 Journal of Computational Mathematics April 1989

COMPUTATION OF A 3–D VISCOUS COMPRESSIBLE NAVIER–STOKES EQUATION FOR SPINNING SHELLS AT MODERATE ANGLES OF ATTACK AND FOR LONG L/D FINNED PROJECTILES

HUANG AI–XIANG
(*Xi'an Jiaotong University, China*)

1. Introduction

The accurate prediction of projectile aerodynamics is of significant importance in the early stage of projectile design.

In recent years, considerable research effort has been focused on the development of modern predictive capabilities for determining projectile aerodynamics, and numerical methods have recently been developed using the 3–D viscous compressible Navier–Stokes computational technique to compute the flow over slender bodies of revolution at transonic or supersonic speeds.

Significant improvement has been made by the author in this paper to make this technique applicable to more complicated flow, by employing finite element methods, the splitting technique of nonlinear operators and the conjugate gradient method for nonlinear subproblems, reduction of an exterior problem into a boundary integral equation, and the domain decomposition method. Applications of the technique are made to a standard shell configuration to establish a benchmark for the code.

2. The Compressible Navier–Stokes Equation in a 3–D Noninertial Coordinate System

We use noninertial curvilinear coordinates $\{x^i\}$ in a rotating reference frame with angular velocity ω. The coordinate axis $z = x^3$ is fixed.

The constitution equation and the dissipation function are given by

$$\tau_{ij} = -pg_{ij} + t_{ij}, \tag{2.1}$$

$$t_{ij}(w) = -\frac{2}{3}\mu_v g_{ij} \text{ div } w + 2\mu_v e_{ij}(w) \tag{2.2}$$

and

$$f_e = -\frac{2}{3}\mu_v(\text{ div } w)^2 + 2\mu_v e^{ij}(w)e_{ij}(w). \tag{2.3}$$

The equations of continuity, momentum and energy for the gas-dynamics in coordinate system $\{x^i\}$ are given by

$$\begin{aligned} &\frac{\partial\rho}{\partial t} + \nabla_i(\rho w^i) = 0, \\ &\rho(\frac{\partial w^i}{\partial t} + w^j\nabla_j w^i + 2\varepsilon^{ijk}\omega_j w_k - (\omega)^2 r^i) = \nabla_j r^{ij} + f^i, \\ &\rho C_v(\frac{\partial T}{\partial t} + w^j\nabla_j T) + p\nabla_j w^j = \text{ div } (k\nabla T) + f_e + h, \\ &\quad p = (r-1)C_v RT \end{aligned} \tag{2.4}$$

where h is heat source per unit volume.

The turbulence models used here are called eddy-viscosity models and are presented for the Navier-Stokes equation (1.1) by expressing μ_v and μ_e in terms of an eddy-viscosity function μ_T, *i.e.*,

$$\mu_v = \mu + \mu_T, \quad \mu_e = r(\frac{\mu}{p_r} + \frac{\mu_T}{p_{r_T}}), \quad r = C_p/C_v, \quad k = C_v\mu_e \tag{2.5}$$

where μ is the molecular viscosity coefficient, μ_T the turbulent viscosity coefficient, μ_e the conductivity, k the conductive coefficient, p_r the Prandtl number, and p_{rT} the turbulent Prandtl number.

The two-equation model employs two additional PDE for variables that are used to define the eddy-viscosity function (K–ε equations). The K–ε equations are given by

$$\text{div } (D_1 \text{ grad } K) = f_1, \quad \text{div } (D_2 \text{ grad } \varepsilon) = f_2$$

where f_1, f_2 are turbulent sources, and the turbulent viscosity is determined by

$$\mu_T = C_v\rho k^2/\varepsilon.$$

We now discuss the boundary conditions. The exterior domain Ω' is decomposed into two regions Ω_s and Ω_2 by a smooth artificial boundary Γ_2. The boundaries of Ω_s consist of the body surface Γ_s and artificial boundary Γ_2. The domain Ω_2 is unbounded. Then,

$$w|_{\Gamma_s} = 0, \quad T|_{\Gamma_s} = 0, \quad w|_{\Gamma_2} = w_\infty, \quad T|_{\Gamma_2} = T_\infty.$$

However, we have another way to treat the artificial boundary condition. In fact, for a far field from body Ω_s, the flow can be assumed to be a potential and imcompressible, nonviscous flow. Let us consider the momentum equation. Because

$$w|_{\Gamma_s} = 0 \text{ and } r^{ij}(w)n_j|_{\Gamma_2} = (-pg^{ij}n_j + 2\mu_v e^{ij}(w)n_j)|_{\Gamma_2} = -pn^i$$