HIGHER ORDER FOLDS IN NONLINEAR PROBLEMS WITH SEVERAL PARAMETERS*

YANG ZHONG-HUA
(*Department of Mathematics, Shanghai University of Science and Technology, Shanghai, China*)

Abstract

In this paper the results in [5] and [6] related to two-parameter nonlinear problems and computing the folds of degree 3 are generalized to any n-parameter nonlinear problems. Constructing a repeatedly extended system for an n-parameter nonlinear problem we prove that a fold of degree $n+1$ corresponds to a regular solution of its n-th extended system. Also, the equivalence between the n-th extended system and its reduced system is proved. Finally, some examples are computed.

1. Introduction

We consider an n-parameter nonlinear problem in the form

$$f(\lambda,\mu_1,\cdots,\mu_{n-1},x)=0 \tag{1.1}$$

where $\lambda,\mu_1,\cdots,\mu_{n-1}\in R, x\in X$, a Banach space, and f is a C^{n+1} mapping from $\underbrace{R\times\ldots\times R}_{n}\times X$ to X.

In many applications some loss of criticality in (1.1), which corresponds to a fold point of higher order at particular values $\lambda^*,\mu_1^*,\cdots,\mu_{n-1}^*$, is concerned. For example, the loss of criticality in the exothermic reaction described by a two-parameter nonlinear problem corresponds to two particular values λ^*,μ^* which are called the third degree fold point of $f(\lambda,\mu,x)=0$ with respect to λ.

In the case $n=2$, following the idea suggested in [2] and [4], Spence and Werner [5] proposed an "extended system" of the original problem, and proved that a third degree fold

* Received November 29, 1986.

point of $f(\lambda,\mu,x)=0$ with respect to λ corresponds to a second degree fold point of the extended system with respect to μ. Yang and Keller [6] further developed a "double extended system" of $f(\lambda,\mu,x)=0$, and pointed out that a third degree fold point of $f(\lambda,\mu,x)=0$ with respect to λ corresponds to a regular solution of the double extended system.

The outline of this paper is as follows. In Section 2, the one-parameter case is discussed and definitions of fold points are given. We introduce some special polynomial operators and discuss their properties. We prove a sufficient and necessary condition for a fold point of higher order.

In Section 3, we discuss the two-parameter case

$$f(\lambda,\mu,x)=0 \tag{1.2}$$

and consider the relation between (1.2) and its extended system. We generalize the results in [5] and prove that a fold point of degree $n+1$ of (1.2) with repect to λ corresponds to a fold point of degree n of its extended system with repect to μ.

Section 4 contains the main results of the paper. We apply the idea of the extended system repeatedly for the n-parameter case

$$f(\lambda,\mu_1,\cdots,\mu_{n-1},x)=0. \tag{1.3}$$

We develop "the n-th extended system" of (1.3) and prove that a fold point of degree $n+1$ of (1.3) corresponds to a regular solution of its n-th extended system. A reduced system for the n-th extended system is introduced in order to compute the fold point of degree $n+1$ practically.

Section 5 contains two numerical examples in which there are folds of degree 3 and degree 4.

2. One–Parameter Case and Fold Points

We consider a one-parameter nonlinear problem in a Banach space X

$$f(\lambda,x)=0 \tag{2.1}$$

where $\lambda\in R, x\in X$ and f is a C^{n+1}($n\geq 1$ is a suitable positive integer) mapping from $\mathbb{R}\times X$ to X.

The notations $f_\lambda(a), f_{\lambda\lambda}(a), f_x(a), f_{xx}(a), f_{\lambda x}(a), f_{xxx}(a),\dots$ are used to denote the partial Frechet-derivatives of f at $a=(\lambda,x)\in\mathbb{R}\times X$. We denote the dual pairing of $x\in X$ and $\psi\in X^*$ by ψx where X^* is the conjugate space of X.