SPACE-TIME TRANSFER FUNCTION-NOISE MODELING OF RAINFALL-RUNOFF PROCESS*

KAZIMIERZ ADAMOWSKI, FADIL B. MOHAMED

(*Department of Civil Engineering, University of Ottawa, Ottawa, Ontario, Canada K1N 6N5*)

NICOLAS R. DALEZIOS

(*INTERA Technologies Ltd., 785 Carling Ave., Ottawa, Ontario, Canada K1S 5H4*)

Abstract

An explanatory model class belonging to the family of Space-Time Transfer Function-Noise (STTFN) processes is presented. The paper develops a three-stage iterative procedure for building STTFN models of the rainfall-runoff process. Four precipitation and runoff stations located in a watershed in southern Ontario, Canada, sampled at 15-day intervals are used for the numerical analysis. Three STTF models are identified. The model parameters are estimated by the polytope technique, a nonlinear optimization algorithm. Two of the developed space-time models proved adequate in describing the spatio-temporal characteristics of precipitation and runoff time series.

§ 1. Introduction

In recent years a large number of stochastic models have been adapted to represent different aspects of the rainfall-runoff process. The most extensively used approach has been the Box-Jenkins (Box and Jenkins, 1976) transfer function-noise (TFN) modeling of hydrologic time series. In particular, linear input-output models are developed with a stochastic part (Strupczewski and Budzianowski, 1984). In this way an increase in accuracy can be achieved within the class of linear models. This modeling procedure relates the output (runoff) of a hydrologic system to the input (rainfall) of the system by adding a noise series. These empirical models have proven very useful in hydrologic analysis and modeling (Salas et al., 1980) and can be used in long-term, as well as short-term hydrologic forecasting.

Besides single-input systems, this class of TFN models has also been used in multi-input systems (Tiao and Box, 1979; Chow et al., 1983). For example, TFN models have been developed relating river flow to precipitation, groundwater levels and temperature. Similarly, the transformation of precipitation series to modified processes has been examined based on cross-correlations of the original and the modified series and by accounting for evaporation and soil moisture storage. Chang et al. (1982) have also developed a TFN model of daily rainfall-runoff process and applied it to five Indiana watersheds. Moreover, a TFN model was developed (Adamowski and Hamory, 1983) relating groundwater levels (output) to streamflow (input).

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There is an increasing interest in hydrology to develop empirical spatio-temporal models of rainfall-runoff in the context of regional hydrologic analysis. Since rainfall and runoff series are correlated in space and time, the Box-Jenkins TFN modeling procedure is extended to a multivariate input-output hydrologic system (Cooper and Wood, 1982; Mohamed, 1985). This results in a general model class of space-time transfer function-noise (STTFN) models. The STTFN model is extended into the spatial domain by using a hierarchical ordering of the spatial neighbors of each rainfall and runoff gage site. The purpose of this study is to develop space-time models of the rainfall-runoff process from the general class of space-time transfer function-noise (STTFN) processes suitable for regional hydrologic analysis and forecasting. In Section 2 the three-stage iterative procedure of identification, parameter estimation and diagnostic checking of the STTFN models is discussed. Section 3 presents an application of the space-time model class to actual rainfall-runoff data for a selected watershed located in southern Ontario, Canada (Fig. 1).

§ 2. Space-Time TFN Model Development

In STTFN modeling, the output y_{it} from $i=1, 2, \cdots, N$ zones over $t=1, 2, \cdots, T$ time periods is assumed to be linearly dependent upon the input series $X_1, X_2, \cdots$, etc. in time and space. The STTFN model may take the form

$$y_{it}=\frac{\sum_{s=0}^{l}\sum_{k=1}^{p}\omega_{sk}B^k L_s}{\left(1-\sum_{s=0}^{m}\sum_{k=1}^{q}\delta_{sk}B^k L_s\right)} B^b L_f x_{it}+a_{it}, \tag{1}$$

where l and m are the spatial orders, p and q are the temporal orders, b and j define an initial period of pure delay or dead time before the response to a given input change begins to take effect, a_{it} is the output noise series independent of x_{it}, B is the backward shift operator in time defined as $B^k Y_{it}=Y_{i(t-k)}$, L_s is the spatial lag operator and ω_{sk} and δ_{sk} are parameters.

The spatial lag operator L_s is defined such that

$$L_s y_{it}=\sum_{j=1}^{N} w_{ijs} y_{jt}, \quad \text{for } s>0, \tag{2}$$

where w_{ijs} are a set of weights scaled so that

$$\sum_{j=1}^{N} w_{ijs}=1 \tag{3}$$

for all i and w_{ijs} nonzero only for i and j sites being sth order neighbors. For $s=0$, equation (2) becomes $L_0 Y_{it}=y_{it}$. The weights follow a hierarchical ordering of spatial neighbors based on distances between the observation sites in the watershed and may reflect physical characteristics of the observed time series.

2.1. Identification of the STTFN Model

The space-time cross-correlation function (STCCF) between y_{it} and x_{it} series at spatial lag s and time lag k is given by