SPACE-TIME ARIMA MODELING FOR REGIONAL PRECIPITATION FORECASTING*

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Abstract

An aggregate regional forecasting model class belonging to the general family of Space-Time Auto Regressive Moving Average (STARMA) processes is investigated. These models are characterised by autoregressive and moving average terms lagged in both time and space. The paper demonstrates an iterative procedure for building a STARIMA model of a precipitation time series. Eleven raingage sites located in a watershed in southern Ontario, Canada, sampled at 15-day intervals for the period of 1966 to 1980 are used in the numerical analysis. The identified model is STMA(l_2). The model parameters are estimated by the polytope technique, a nonlinear optimization algorithm. The developed model performed well in regional forecasting and in describing the spatio-temporal characteristics of the precipitation time series.

§ 1. Introduction

Time series analysis for modeling and forecasting of hydrologic variables is a valuable and important step in water resources planning and management. In hydrology, the selection of models for analysis of time series is essentially based on simulation and statistical decision theory (Salas et al., 1980). A flexible class of empirical models is the general family of autoregressive moving average (ARMA) processes (Box and Jenkins, 1976). These models have proven very useful in hydrologic analyses (Hipel et al., 1977), but since they are univariate, they are applicable only to single series of data. In constructing an appropriate dynamic stochastic model for a given one-dimensional time series a three-stage iterative procedure is usually followed. This is commonly referred to as the Box-Jenkins method (Box and Jenkins, 1976).

There is an increasing interest in hydrology to develop empirical spatio-temporal models in the context of hydrologic regional analysis and forecasting (Perry and Aroian, 1979; Pfeifer and Deutsch, 1981; Mohamed, 1985). Naturally, an alternative to univariate time series modeling is multivariate time series modeling (Anderson, 1958; Hannan, 1970). An appropriate class of formal models for describing space-time hydrologic data sets is again provided by linear stochastic difference equations. These models attempt to simultaneously describe a set of N

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observable time series. When these N time series represent spatially-located data, the interrelationships and the spatial correlation (Cliff and Ord, 1973) between the different spatial data sets can be taken into account and thus a better system description should result. The spatial domain is incorporated in the modeling procedure by using a hierarchical ordering of the spatial neighbors of each location (Besag, 1974).

The objective of this paper is to develop a spatio-temporal precipitation time series model from the general class of space-time autoregressive moving average (STARMA) processes suitable for regional hydrologic analysis and forecasting. The methodology is essentially an extension of the Box-Jenkins model-building procedure to take into account the spatial effect of existing time series over hydrologically homogeneous areas. In other words, the watershed is considered to be homogeneous with respect to the spatial and temporal variation of physical and hydrologic characteristics, climatic variables and system response.

The paper is organized as follows. In Section 2 the three-stage iterative model-building procedure is developed. Specifically, in the identification stage a preliminary analysis of the data is performed to select a tentative spatio-temporal model. The order of the STARMA model is chosen based on the estimation of space-time autocorrelation (STACF) and space-time partial autocorrelation functions (STPACF). The second stage covers the parameter estimation of the selected tentative STARMA model. The third stage of diagnostic checking deals with the adequacy of the fitted STARMA model, since the goal remains to obtain an adequate but parsimonious model with the smallest number of parameters meeting certain statistical accuracy criteria. If any inadequacy is found, the three-stage iterative procedure is repeated. Section 3 describes the data environment and, finally, in Section 4 a hydrologic application is discussed.

§ 2. Starima Model-Building Procedure

An extension of the Box-Jenkins univariate ARIMA process into the spatial domain leads to the formulation of the general family of STARIMA models (Martin and Oeppen, 1975; Cliff and Orb, 1975; Bennett, 1975; Hooper and Hewings, 1981; Pfeifer and Deutsch, 1980; Mohamed, 1985). The general form of the STARMA model is

$$y_{it}=a_{it}+\sum_{s=0}^{l}\sum_{k=1}^{p}\phi_{sk}L_s y_{i(t-k)}-\sum_{s=0}^{m}\sum_{k=1}^{q}\theta_{sk}L_s a_{i(t-k)}, \tag{1}$$

where y_{it} is the time series at time t and at site i, $i=1, 2, \cdots, N$, p and q are the temporal orders of the AR and MA terms, respectively, l and m are the spatial orders of the AR and MA terms, respectively, ϕ_{sk} and θ_{sk} are parameters of the AR and MA terms, respectively, with spatial lag s and temporal lag k, L_s is the spatial lag operator of lag s, and a_{it} are normally independently distributed white noise residuals with

$$E[a_{it}]=0$$

and

$$E[a_{it}a_{j(t-k)}]=\begin{cases}\sigma^2, & \text{for } i=j,\ k=0,\\ 0, & \text{otherwise.}\end{cases} \tag{2}$$