ON COLLOCATION METHODS FOR SOLVING THE NEUTRON TRANSPORT EQUATION IN TWO-DIMENSIONAL PROBLEMS*

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Abstract

Collocation methods are considered for solving the time-dependent neutron transport equation in two-dimensional planar geometry. Error estimates and stability are derived. Finally, some numerical results are presented.

Introduction

The neutron transport equation is an integral-differential equation in which the differential part is of hyperbolic type. In solving a neutron transport equation[1], can be as simple and convenient as the DSN method[2], and the logical construction of the program is generally the same except that a lower degree linear algebraic system must be solved on each mesh. In fact, the DSN method is a special type of collocation method. It is a weighted residual method and is equivalent to the discrete Galerkin method. The collocation methods have higher accuracy and faster convergence and requires less operating time to attain the same accuracy than the DSN method.

Many authors have done works of value in using collocation methods to solve partial differential equations, for example[3], [4].

In this paper, we will use the collocation method to solve the time-dependent neutron transport equation in the two-dimensional x, y-plane geometry. Here, the Crank-Nicholson central difference is used to approximate the time variable, and the discrete ordinates approximation is used for the angular variables. An outline of the paper is as follows: the calculation method is given in Section 1. In Sections 2—4, error estimates and stability are derived. In Section 5, we discuss conservation of the method and some relations, such as its comparison with the difference method and the discrete Galerkin method. Finally, in order to explain the effectiveness of the methods, some numerical results are presented.

§ 1. Numerical Method

For the sake of simplicity we consider the initial-boundary value problem for the one-group neutron transport equation as follows:

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$$\begin{cases} A(\Phi) \equiv \frac{1}{v^*}\frac{\partial \Phi}{\partial t} + \mu\frac{\partial \Phi}{\partial x} + \nu\frac{\partial \Phi}{\partial y} + \alpha\Phi = Q(\Phi) + F, \\ \Phi(t, x, y, \mu, \nu)|_{t=0} = \Phi_0(x, y, \mu, \nu), \\ \Phi(t, x, y, \mu, \nu)|_{\Gamma} = 0 \quad \text{for} \quad \boldsymbol{\Omega}\cdot\boldsymbol{h} \leqslant 0, \\ \Phi(t, x, y, \mu, \nu)|_{x=0} = \Phi(t, x, y, -\mu, \nu)|_{x=0}, \\ \Phi(t, x, y, \mu, \nu)|_{y=0} = \Phi(t, x, y, \mu, -\nu)|_{y=0}, \end{cases} \tag{1.1}$$

where the function $\Phi(t, x, y, \mu, \nu)$ represents the angular flux of neutrons at the point (t, x, y) and the angular direction $\boldsymbol{\Omega} = (\mu, \nu)$, where

$$\mu = \sin\theta\cos\psi,$$
$$\nu = \sin\theta\sin\psi,$$
$$\xi = \cos\theta.$$

Thus the neutron velocity $\boldsymbol{v}^* = \boldsymbol{v}^*\boldsymbol{\Omega}$ (see Fig. 1). Here, α, β are some nuclear data, and are supposed to be block constants, satisfying

$$0 < \alpha_0 \leqslant \alpha \leqslant \alpha_1, \quad 0 < \beta_0 \leqslant \beta \leqslant \beta_1.$$

Denote by B the region in which to solve equation (1.1)

$$B = D_t \times D \times D_{\mu\nu},$$

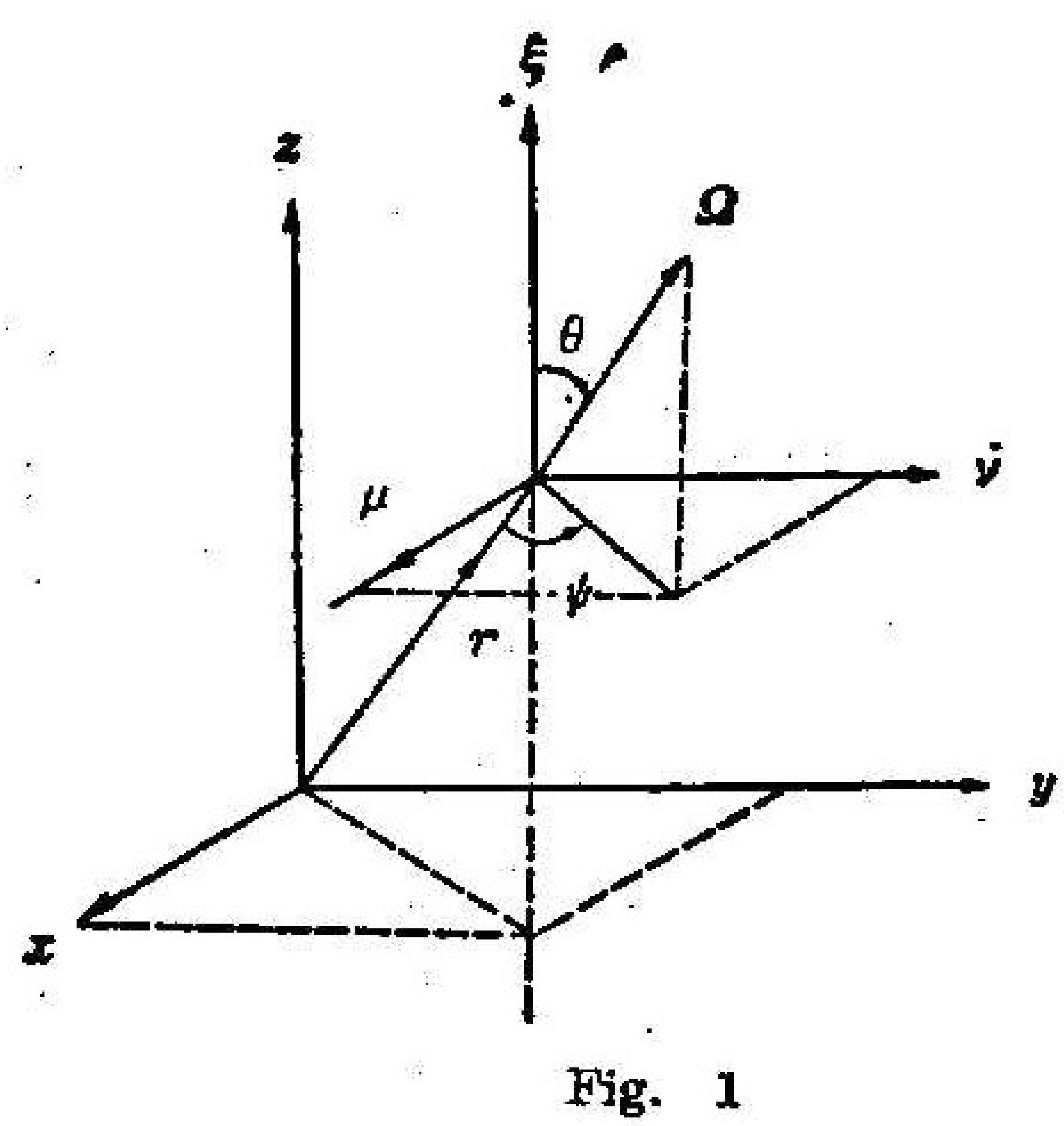

Fig. 1

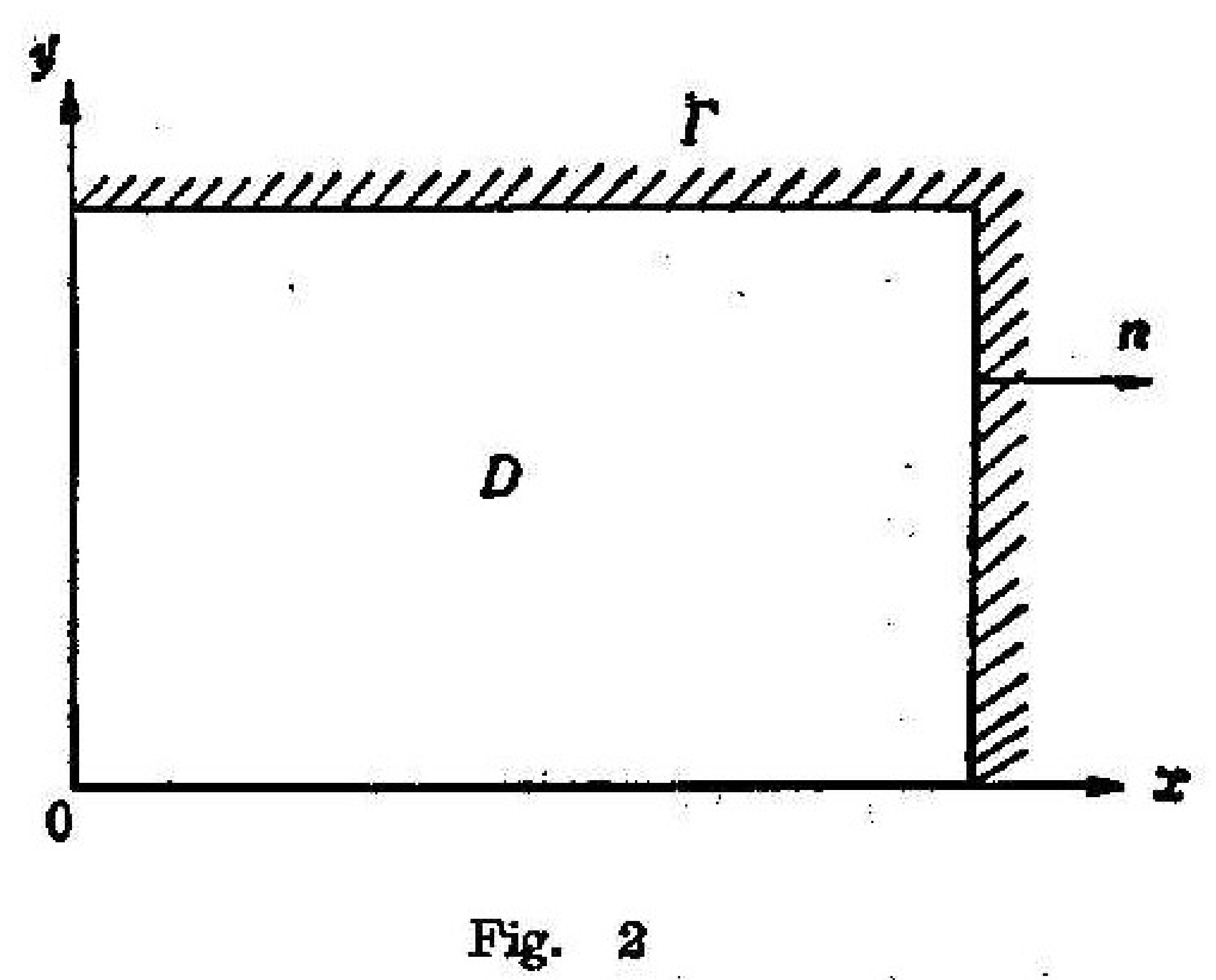

Fig. 2

where D_t: $[0, T]$, $D = D_x \times D_y$, D_x: $[0, X]$, D_y: $[0, Y]$, $D_{\mu\nu}$ is the unit disk in the (μ, ν)-plane: $\mu^2 + \nu^2 \leqslant 1$. Γ is the boundary of D. Denote by $\boldsymbol{n}$ the unit vector in the direction of outward normal to Γ (see Fig. 2).

$$Q(\Phi) = \beta\int_{\Omega} \Phi \, d\Omega.$$

An outside source term is denoted by $F(t, x, y, \mu, \nu)$. Suppose that Φ_0 is continuous on $D \times D_{\mu\nu}$.

We divide the spatial variables and time into

$$0 = x_0 < x_1 < \cdots < x_I = X;$$
$$0 = y_0 < y_1 < \cdots < y_J = Y;$$
$$0 = t_0 < t_1 < \cdots < t_N = T.$$