ON ERROR ESTIMATE OF THE BOUNDARY ELEMENT METHOD FOR PARABOLIC EQUATIONS IN A TIME-DEPENDENT INTERVAL*

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Abstract

This paper discusses the direct boundary element method for parabolic equations in a time-dependent interval. An optimal estimate of the error in maximum norm for the boundary element collocation scheme is given.

§ 1. Introduction

Compared with the domain methods such as the finite difference method or the finite element method, the boundary element method reduces the dimensions of the problem by one, so that the amount of computational work can be greatly decreased. In recent years, therefore, some authors studied its applications to numerical solution of parabolic equations and moving boundary problems (e.g. [1]—[3]). However, little work on mathematical analysis of the convergence of the method has been done. The only published work, to the author's knowledge, is by K. Onishi ([4]). But, as pointed out by the author in [5], his proof is based on a wrong estimate of matrix norm and thus is incorrect. In [5], the author proved the uniform convergence of the boundary element method and gave an optimal error estimate in maximum norm for the one-dimensional heat equation, using the method of matrix analysis which is not, however, applicable to problems in a two-dimensional or time-dependent domain.

In this paper we give an optimal estimate of the error for the boundary element collocation scheme for heat equation in a time-dependent interval, using the theory of operator analysis. The two-dimensional case will be discussed in another paper.

§ 2. Parabolic Equation in a Time-dependent Interval

For definiteness, we consider the following heat equation:

$$k\frac{\partial^2 u}{\partial x^2}-\frac{\partial u}{\partial t}=0,\quad 0<x<S(t),\ 0<t\leqslant T<\infty, \tag{2.1}$$

* Received May 11, 1985.

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$$\frac{\partial u}{\partial x}(0, t)=q_1(t), \quad \frac{\partial u}{\partial x}(S(t), t)=q_2(t), \quad 0<t\leqslant T, \tag{2.2}$$

IC. $$u(x, 0)=u_0(x), \quad 0\leqslant x\leqslant S(0) \quad (S(0)=L>0), \tag{2.3}$$

where $k>0$ is a constant, $q_i(t)\,(i=1, 2)$ are bounded, $u_0(x)$ is Lipschitz continuous, $S(t)$ has continuous first derivative $\dot{S}(t)$ and is assumed, without loss of generality, to be a nondecreasing function of t.

§ 3. Boundary Integral Equation

The fundamental solution of (2-1) is

$$u^*(x, \xi; t, \tau)=\frac{1}{2\sqrt{\pi k(t-\tau)}}\exp\left[-\frac{(x-\xi)^2}{4k(t-\tau)}\right], \quad t>\tau. \tag{3.1}$$

Let

$$q^*(x, \xi; t, \tau)=\frac{\partial u^*}{\partial \xi}=\frac{(x-\xi)}{4\sqrt{\pi}\,(k(t-\tau))^{3/2}}\exp\left[-\frac{(x-\xi)^2}{4k(t-\tau)}\right], \quad t>\tau, \tag{3.2}$$

$$g_1(t)=2\left\{-k\int_0^t q_1(\tau)u^*(0, 0; t, \tau)d\tau+k\int_0^t q_2(\tau)u^*(0, S(\tau); t, \tau)d\tau \right.$$
$$\left. +\int_0^L u_0(\xi)u^*(0, \xi; t, 0)d\xi\right\}, \tag{3.3a}$$

$$g_2(t)=2\left\{-k\int_0^t q_1(\tau)u^*(S(t), 0; t, \tau)d\tau+k\int_0^t q_2(\tau)u^*(S(t), S(\tau); t, \tau)d\tau \right.$$
$$\left. +\int_0^L u_0(\xi)\, u^*(S(t), \xi; t, 0)d\xi\right\}. \tag{3.3b}$$

The boundary integral equation corresponding to (2.1)—(2.3) is ([1])

$$\frac{1}{2}u(0, t)-k\int_0^t q^*(0, 0; t, \tau)u(0, \tau)d\tau+k\int_0^t q^*(0, S(\tau), t, \tau)u(S(\tau), \tau)d\tau$$
$$-\int_0^t u^*(0, S(\tau), t, \tau)\dot{S}(\tau)u(S(\tau), \tau)d\tau=\frac{1}{2}g_1(t), \tag{3.4a}$$

$$\frac{1}{2}u(S(t), t)-k\int_0^t q^*(S(t), 0; t, \tau)u(0, \tau)d\tau$$
$$+k\int_0^t q^*(S(t), S(\tau); t, \tau)u(S(\tau), \tau)d\tau$$
$$-\int_0^t u^*(S(t), S(\tau); t, \tau)\dot{S}(\tau)u(S(\tau), \tau)d\tau=\frac{1}{2}g_2(t), \tag{3.4b}$$

Define the column vectors $U(t)=(u_1(t), u_2(t))^T$ with $u_1(t)=u(0, t)$, $u_2(t)=u(S(t), t)$, $G(t)=(g_1(t), g_2(t))^T$ and the matrix $K(t, \tau)=(k_{ij}(t, \tau))_{2\times 2}$ s.t.

$$\begin{cases} k_{ij}=0, \; 1\leqslant i, j\leqslant 2, \; 0\leqslant t<\tau\leqslant T; \\ k_{11}=q^*(0, 0; t, \tau), \quad k_{12}=u^*(0, S(\tau); t, \tau)\dot{S}(\tau)-q^*(0, S(\tau); t, \tau), \\ k_{21}=q^*(S(t), 0; t, \tau), \quad k_{22}=u^*(S(t), S(\tau); t, \tau)\dot{S}(\tau)-q^*(S(t), S(\tau); t, \tau), \\ \qquad 0\leqslant \tau<t\leqslant T. \end{cases} \tag{3.5}$$

Let $\lambda=2k$. The boundary integral equation can, then, be written in the form