QUANTITATIVE COMPARISON AMONG SEVERAL DIFFERENCE SCHEMES*1)

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Abstract

When one compares a difference scheme with another, only a qualitative comparison is usually given. Such a comparison is not enough. For example, scheme A is more accurate than scheme B, but it would take more time to use scheme A than to use scheme B. In order to determine which scheme is the best, it is necessary to make a quantitative comparison among difference schemes.

When the state equation is non-convex, the numerical solution is sensitive to methods[1,2]. We make a numerical test with 10 different schemes for such a problem. From the computed results the following conclusions are obtained:

The physically relevant solutions can be obtained if the Godunov scheme and the first-order E-O scheme are used, but the solutions are not so accurate. In our problem, it is necessary to take at least 80000 mesh points in the space direction in order to obtain a solution with an error of 10^{-3}. The physically relevant solution can also be obtained by using the Lax scheme, but its accuracy is lower than those of the Godunov scheme and the E-O scheme.

The physically relevant solutions cannot be obtained by using the L-W scheme, the MacCormack scheme, the Murman scheme, the Richtmyer scheme, the Courant scheme and the second-order one-sided scheme.

The physically relevant solutions can also be obtained by using the second-order singularity-separating method (S-S scheme for short). For our problem, 15—25 mesh points in the space direction are enough for a solution whose error is 10^{-3}. That is, in this case the number of mesh points for the S-S method is $\frac{1}{3200}$—$\frac{1}{5300}$ of that for the Godunov scheme or the first-order E-O scheme. We know from the computation that the convergence rates of the Godunov scheme and the first-order E-O scheme are about $O(\Delta t^{1/2})$ in L_2 space, but that of the S-S method is $O(\Delta t^2)$ 2). We can see that the higher the required accuracy, the larger the difference of the computation amount. Moreover, because of the rounding errors, we cannot make the computational error infinitely small. If we solve our problem using a computer with a word length of 32 bits (the length of mantissa is 24 bits), the smallest possible error is 7×10^{-3} for the E-O scheme, but it is 10^{-5} for the S-S method. This is because the amount of computation for our method is less, and the problem of rounding errors is not so serious.

§ 1. The Problem

We consider the following initial-boundary-value problem with a non-convex equation of state:

$$\begin{cases} \dfrac{\partial U}{\partial t}+\dfrac{\partial f(U)}{\partial x}=0, \\ U(x,0)=\begin{cases} 0.656-200(x+0.001), & -0.001\leqslant x<-0.0005, \\ 0.656+200x, & -0.0005\leqslant x<0, \\ 0.014+170x, & 0\leqslant x<0.0005, \\ 0.014-170(x-0.001), & 0.0005\leqslant x\leqslant 0.001. \end{cases} \end{cases}$$

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2) In § 4 the definitions of norms are given.

$$\begin{cases} U(-0.001,\ t)=0.656-4x_1(t), \\ U(0.001,\ t)=0.014-3.4x_2(t), \end{cases}$$

where the equation of state is

$$f(U)=U^4/2-19U^3/30+U^2/4-33U/1000,$$

and $x_1(t)$, $x_2(t)$ are implicitly given by the following formulas

$$x_1(t)=tf'(0.656-4x_1(t)),$$
$$x_2(t)=tf'(0.014-3.4x_2(t)).$$

§ 2. The Schemes

The following schemes are used for the above problem.

1. The singularity-separating method (the S-S method)[8,16]

This method has been proven both in theory and in practice to be a very good method for the initial-boundary-value problem of the first-order quasi-linear hyperbolic equations. The discontinuity conditions are used on the discontinuity lines and there is no difference across the discontinuity lines in the system of difference equations. More accurate solutions can be obtained by using a few mesh points because of the small truncation errors. The figure of the solution for our problem is shown in Fig. 1. In order to solve this problem by the S-S method, we introduce a new coordinate system through the following coordinate transformation:

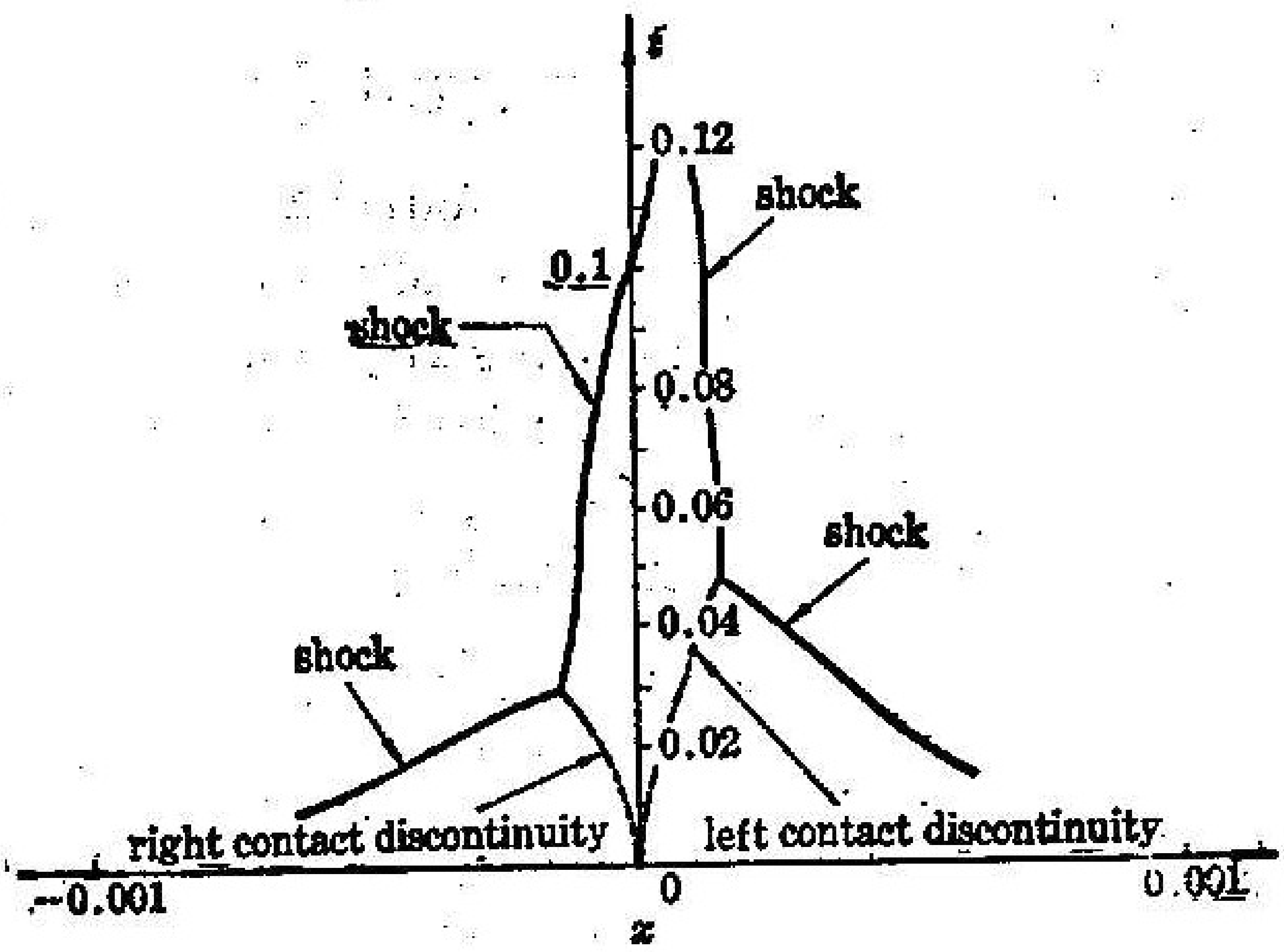

Fig. 1 Locations of the discontinuities of $U(x,t)$ in plane (x,t) (the S-S method)

$$\begin{cases} \xi=\dfrac{x-x_{l-1}(t)}{x_l(t)-x_{l-1}(t)}+l-1, & \text{if } x_{l-1}(t)\leqslant x\leqslant x_l(t), \\ t=t. \end{cases}$$

Here $x_l(t)$ represents a boundary line or a discontinuity line (From Fig. 1 it is clear that there are several discontinuity lines).

Through the above transformation, a problem with movable boundaries is changed into a problem with fixed boundaries. It is convenient for treating boundaries accurately. Suppose

$$\lambda=\frac{\partial\xi}{\partial t}+\frac{\partial f}{\partial U}\frac{\partial\xi}{\partial x},$$

then the equation in the new coordinate system can be written as:

$$\frac{\partial U}{\partial t}+\lambda\frac{\partial U}{\partial\xi}=0.$$

For this equation the following scheme is used in each subregion. Suppose