# A NEW VERSION OF ITERATIVE METHOD FOR SOLVING RIEMANN PROBLEM*

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## Abstract

A new version of iterative method for solving Riemann problem of gas dynamics is presented. In practice the new procedure exhibited a good convergence in cases where Riemann solution involves a strong rarefaction wave or two rarefaction waves. In the other cases the new version is identical with Godunov procedure.

## Introduction

Riemann solutions are the building blocks of several numerical methods for solving the equations of gas dynamics (see [1], [3], [4], [7]). The usefulness of these methods depends on the possibility of solving the Riemann problem accurately and effectively. Generally speaking, Godunov iterative procedure provides an approximating solution to the Riemann problem ([3], [5]). But as noted by Godunov, the iteration may fail to converge in the presence of strong rarefaction. To overcome this difficulty Chorin gave a modified iterative method [1]. In this paper we present a new version of iterative method. In practice we find that the new version is more effective in cases where the Riemann solution consists of two rarefactions or a rarefaction plus a shock where the rarefaction is stronger than the shock. In the other cases the new version is identical with Godunov procedure.

## The New Version of Iterative Procedure

Consider the gas dynamics equations

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p)_x = 0, \\ e_t + ((e+p)u)_x = 0, \end{cases} \tag{1}$$

where $\rho$, $u$, $\rho u$, $e$ and $p$, respectively, denote the density, velocity, momentum, internal energy, and pressure of the gas, and

$$e = p\varepsilon + \frac{1}{2}\rho u^2$$

is the total energy of the gas. For polytropic gases $\varepsilon$ is given by the constitutive relation

$$\varepsilon = \frac{1}{\gamma - 1}\frac{p}{\rho},$$

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where $\gamma$ is a constant larger than one.

The Riemann problem for (1) will have the initial data

$$(\rho(x,0),\ u(x,0),\ p(x,0))=\begin{cases} S_l=(\rho_l,\ u_l,\ p_l), & x<0,\\ S_r=(\rho_r,\ u_r,\ p_r), & x>0.\end{cases}$$

It is well-known that the solution consists of a right state $S_r$, a left state $S_l$, a middle state $S_*(p=p_*,\ u=u_*)$, separated by waves which are either rarefactions or shocks. $S_*$ is divided by the slip line

$$\frac{dx}{dt}=u_*$$

into two parts with possibly different values of $\rho_*$, but equal values of $u_*$ and $p_*$.

The new version of procedure is as follows.

1. In case of $u_l<u_r$, the iterative method first computes $u_*$ in the state $S_*$. Define the quantity

$$M_r=(p_r-p_*)/(u_r-u_*). \tag{2}$$

The relation between $S_*$ and $S_r$ can be written ([2],[7]) as

$$u_*=u_r+\varphi(p_*;\ p_r,\ \rho_r), \tag{3}$$

where
$$\varphi(p_*;\ p_r,\ \rho_r)=\begin{cases}\dfrac{\sqrt{2}\,(p_*-p_r)}{\sqrt{((\gamma+1)p_*-(\gamma-1)p_r)\rho_r}}, & p_*\geqslant p_r,\\[2ex] \dfrac{2\sqrt{\gamma}}{\gamma-1}\,\dfrac{p_r^{1/2\gamma}}{\rho_r^{1/2}}\left(p_*^{(\gamma-1)/2\gamma}-p_r^{(\gamma-1)/2\gamma}\right), & p_*<p_r.\end{cases}$$

Upon solving $p_*$ in terms of $u_*$ from (3), we have

$$p_*=p_r+\psi(u_*-u_r;\,p_r,\ \rho_r), \tag{4}$$

where
$$\psi(u_*-u_r;\ p_r,\rho_r)=\begin{cases}\dfrac{\gamma+1}{4}(u_*-u_r)^2\rho_r\left(1+\sqrt{1+\dfrac{16\gamma p_r}{(\gamma+1)^2(u_*-u_r)^2\rho_r}}\right), & u_*\geqslant u_r,\\[2ex] p_r\left(\left(1+\dfrac{\gamma-1}{2\sqrt{\gamma}}(u_*-u_r)\sqrt{\rho_r/p_r}\right)^{\frac{2\gamma}{\gamma-1}}-1\right), & u_*<u_r.\end{cases}$$

By substituting $p_*$ of (4) into (2), one gets

$$M_r=(p_r\rho_r)^{1/2}\Psi((u_*-u_r)(\rho_r/p_r)^{1/2}). \tag{5}$$

where
$$\Psi(x)=\begin{cases}\dfrac{\gamma+1}{4}\left(x+\left(x^2+\dfrac{16\gamma}{(\gamma+1)^2}\right)^{1/2}\right), & x\geqslant 0,\\[2ex] \dfrac{1}{x}\left(\left(1+\dfrac{\gamma-1}{2\sqrt{\gamma}}x\right)^{\frac{2\gamma}{\gamma-1}}-1\right), & x<0.\end{cases}$$

Similarly, $M_l$ is defined by

$$M_l=-(p_l-p_*)/(u_l-u_*) \tag{6}$$

The relation between $S_*$ and $S_l$ is

$$u_*=u_l-\varphi(p_*;\ p_l,\ \rho_l), \tag{7}$$

or

$$p_*=p_l+\psi(u_l-u_*;\ p_l,\ \rho_l). \tag{8}$$

From (6) and (7), one gets

$$M_l=(p_l\rho_l)^{1/2}\Psi((u_l-u_*)(\rho_l/p_l)^{1/2}). \tag{9}$$