THE QUADRATIC COLLISION PROBABILITY METHOD AND THE IMPORTANCE SAMPLING METHOD IN MONTE CARLO CALCULATION FOR THE FLUX AT A POINT*

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Abstract

The unbounded estimate is one of the troublesome problems in Monte Carlo method. Particularly, in the calculation for the flux at a point, the estimate may approach infinite. In this paper, a collision probability method is proposed in Monte Carlo calculation for the flux at a point, and two kinds of methods with the bounded estimation are presented: the quadratic collision probability method and the importance sampling method. The former method is simple and easy to use, whereas the latter is suitable for calculation of flux at many different points simultaneously. The practical calculation indicates that the variance of the present methods can be reduced by about 50 percent and the efficiency can be increased by 2 to 4 time in comparison with the existing methods.

1. Introduction

The application of Monte Carlo method to the calculation for the flux at a point plays an important part in the particle transport problems. It is because, first of all, the calculation of the point flux is often encountered in the practical problems. Second, because the problem of any local flux calculation can be solved through the calculation for the flux at a point. Finally, there are some difficulties with numerical calculation. Particularly, the problem is more serious for those problems with complicated geometry and other factors.

Let $\varphi(\boldsymbol{r}^0)$ denote the flux at the point $\boldsymbol{r}^0$. In other words, $\varphi(\boldsymbol{r}^0)d\boldsymbol{r}^0$ is the average track length by the particle through the volume element $d\boldsymbol{r}^0$ near $\boldsymbol{r}^0$. Thus, in order to calculate the point flux $\varphi(\boldsymbol{r}^0)$ by the usual Monte Carlo method, it is necessary to choose such a geometric volume V containing $\boldsymbol{r}^0$ in it that the $\varphi(\boldsymbol{r}^0)$ can be approximately obtained as follows:

$$\varphi(\boldsymbol{r}^0)\approx\varphi(V)=\int_V \varphi(\boldsymbol{r})d\boldsymbol{r}/|V|,$$

where $|V|$ is the volume of the geometric region. In order to make the approximate equation $\varphi(\boldsymbol{r}^0)\approx\varphi(V)$ hold more exactly, the geometric region V is taken very small. Thus, the general Monte Carlo method becomes very difficult.

There are probably two different ways to overcome the difficulty mentioned above. One is to exchange the location between the particle source and the detector (the point flux response function) so that the original particle source is turned into

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the detector, while the original detector into the particle source. In this way, the problem with the point flux calculation can be changed into one with the non-point flux calculation as long as the original source is not a point source. In this aspect, there are the reciprocal Monte Carlo[1,2] and the adjoint Monte Carlo[3-5]. Another is to use the statistical estimation techniques to treat the variables analytically involved where possible, or to use the biased sampling techniques to treat the variables which can cause large fluctuations in the results. There are the directing probability method[6-8], biased location sampling method[7], the maximum cross section method[9] and the reselection method[10-13] and so on.

The reciprocal Monte Carlo method has two important disadvantages. One is that the particle source can not be a point source. Another, the application is extremely limited, because the condition under which the source and the detector can be exchanged is seldom satisfied. As for the adjoint Monte Carlo the situation is different. In this case, the reciprocity between the source and detector is done formally, and does not meet the condition which is needed in the real reciprocity. Therefore, it overcomes the second difficulty appearing in the reciprocal Monte Carlo method successfully. But the adjoint Monte Carlo does not remove the first disadvantage, nevertherless, and it creates some new problems, such as the complicated random walk, larger statistical error[14] and so on.

The directing probability method is a very simple and easy to use. But if there is scattering medium near point $\boldsymbol{r}_0$, the estimation of the directing probability is unbounded. For the homogeneous medium, Kalos proved that the estimation of the directing probability method is not only unbounded, but also its variance is divergent (for the heterogeneous medium, as long as there is some scattering medium near the point $\boldsymbol{r}^0$, the variance of the directing probability method is divergent as well). The boundlessness of the estimation often makes the statistical fluctuation of the Monte Carlo estimate become large. Meanwhile the divergence of the variance can directly affect the convergence rate.

The location biased sampling method which was given by Kalos in 1963 is the first method with finite variance to overcome the divergence of the directing probability method. But his conclusion is based on the assumption of homogeneous medium, monoenergetic particle and isotropic scattering. On the other hand, not only is the method approximate, also it is complicated to apply to the calculation for the flux at a point, hence the method has not been widely used. The maximum cross section method given by Михайлов is another one to solve the variance divergence problem. It does not need any condition which must be satisfied by the location biased sampling method. As the location biased sampling method, although they have solved the problem about the variance divergence, they can not overcome the problem about the unbounded estimate at last.

The reselection method which was developed by Steinberg and Kalos in 1971 is the first one to solve the problem about the boundlessness of the estimate in Monte Carlo calculation for the flux at a point. Later, it was improved further by Steinberg, Lichtenstein, Kalli and Cashwell, but it is still complicated. And it is not able to calculate the flux at many different points simultaneously. In this paper, we present a collision probability method in Monte Carlo calculation for the flux at a point. Based on this, two kinds of the bounded estimate methods are given, namely,