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On [p, q]-order of Solutions of Higher Order Complex Linear Differential Equations in an Angular Domain of Unit Disc

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Abstract. We study the growth of solutions of higher order complex linear differential equations in an angular domain of the unit disc instead of the whole unit disc. Some estimations of $[p,q]$-order of solutions of the higher order differential equations in an angular domain are found in this paper.

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1 Introduction and main results

For a function f meromorphic in the unit disc $\Delta=\{z:|z|<1\}$, the order of growth is given by

$$\rho(f)=\limsup_{r\to 1^-}\frac{\log^+ T(r,f)}{\log\frac{1}{1-r}}.$$

If f is an analytic function in Δ, then the order of growth of f is often given by

$$\rho_M(f)=\limsup_{r\to 1^-}\frac{\log^+\log^+ M(r,f)}{\log\frac{1}{1-r}},$$

where

$$M(r,f)=\max_{\substack{|z|=r\\ z\in\Delta}}|f(z)|,\quad \log^+ x=\max\{\log x,0\}.$$

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It follows from the following inequality in [20, Theorem V.13]

$$T(r,f)\leq \log^{+} M(r,f)\leq \frac{1+3r}{1-r}T\left(\frac{1+r}{2},f\right), \quad r\in(0,1),$$

that

$$\rho(f)\leq \rho_M(f)\leq \rho(f)+1.$$

It is possible that there exists f such that $\rho(f)\neq\rho_M(f)$; for example, $f(z)=\exp\{(\frac{1}{1-z})^{\lambda}\}$ satisfies $\rho(f)=\lambda-1$ and $\rho_M(f)=\lambda$, where $\lambda>1$ is a constant, which can be found in [20, p. 205].

In order to state our results, some notations are needed. For any $r\in(0,\infty)$, $\exp_1 r=\exp r$, $\exp_{n+1} r=\exp(\exp_n r)$, $\log_1 r=\log r$, $\log_{n+1} r=\log(\log_n r)$, $n\geq 1$ is integer. $\exp_0(r)=r=\log_0 r$, $\exp_{-1} r=\log_1 r$. Second, we recall some definitions.

Definition 1.1 ([10])**.** *For f meromorphic in Δ, set*

$$D(f)=\limsup_{r\to 1^-}\frac{T(r,f)}{\log\frac{1}{1-r}}.$$

If $D(f)=\infty$, we say that f is admissible. If $D(f)<\infty$, we say that f is non-admissible.

For the function of fast growth in Δ, we also need the definition of iterated $p-$order, which can be found in [4].

Definition 1.2. *Let f be a meromorphic function in Δ. Then*

$$\rho_p(f)=\limsup_{r\to 1^-}\frac{\log_p^{+} T(r,f)}{\log\frac{1}{1-r}},$$

where $p\geq 1$ is integer. If f is an analytic function in Δ, then the iterated p-order is also given by

$$\rho_{M,p}(f)=\limsup_{r\to 1^-}\frac{\log_{p+1}^{+} M(r,f)}{\log\frac{1}{1-r}}.$$

Obviously, $\rho_1(f)\leq\rho_{M,1}(f)\leq\rho_1(f)+1$ for any analytic functions in Δ. However, it follows from [20, Theorem V.13] that $\rho_p(f)=\rho_{M,p}(f)$ for $p\geq 2$. In general, $\rho_2(f)$ or $\rho_{M,2}(f)$ are called hyper-order of f in Δ. In this paper, we assume that the reader is familiar with the fundamental results and standard notation of the Nevanlinna's theory of meromorphic functions in Δ, see [15] and [25] for more details.

Definition 1.3 ([2, 3])**.** *Let $1\leq q\leq p$ or $2\leq q=p+1$, and f be a meromorphic function in Δ. Then the $[p,q]$-order of f is defined as*

$$\rho_{[p,q]}(f)=\limsup_{r\to 1^-}\frac{\log_p^{+} T(r,f)}{\log_q\frac{1}{1-r}}.$$