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Stochastic Domain Decomposition for Time Dependent Adaptive Mesh Generation

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Abstract. The efficient generation of meshes is an important component in the numerical solution of problems in physics and engineering. Of interest are situations where global mesh quality and a tight coupling to the solution of the physical partial differential equation (PDE) is important. We consider parabolic PDE mesh generation and present a method for the construction of adaptive meshes in two spatial dimensions using stochastic domain decomposition that is suitable for an implementation in a multi– or many–core environment. Methods for mesh generation on periodic domains are also provided. The mesh generator is coupled to a time dependent physical PDE and the system is evolved using an alternating solution procedure. The method uses the stochastic representation of the exact solution of a parabolic linear mesh generator to find the location of an adaptive mesh along the (artificial) subdomain interfaces. The deterministic evaluation of the mesh over each subdomain can then be obtained completely independently using the probabilistically computed solutions as boundary conditions. A small scaling study is provided to demonstrate the parallel performance of this stochastic domain decomposition approach to mesh generation. We demonstrate the approach numerically and compare the mesh obtained with the corresponding single domain mesh using a representative mesh quality measure.

AMS subject classifications: 65N50, 65M50, 65L50, 65C05,65N55, 65M55

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1 Introduction

The numerical solution of many partial differential equations (PDEs) benefits from the construction of an adaptive grid automatically tuned by the solution itself. The quasi–

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Lagrangian (QL), r–refinement, approach used here keeps the number of mesh points and the mesh topology fixed, moving the mesh continuously in time using a moving mesh PDE (MMPDE). The solution of the mesh PDE gives a continuous mesh transformation between an underlying computational co-ordinate and the required physical co-ordinate. Both the mesh and the solution are obtained at each time step. The QL approach can be implemented in either an alternating or simultaneous manner. The simultaneous QL approach treats the MMPDE and physical PDE as one large coupled system. At each time the new mesh and new solution on that mesh are found concurrently. Hence the mesh reacts instantly to changes in the physical solution. This highly nonlinear coupling may destroy exploitable structure which exists in the discretization of the physical PDE alone. The alternating approach uses the current mesh and physical solution to update the mesh alone, this new mesh then facilitates the computation of the updated physical solution. This introduces a time lag in the mesh as the new mesh is based only on the current physical solution. Computationally, however, this decoupling reduces the size of the discrete problem. Furthermore, the solver becomes more modular; the mesh and physical solvers can be called in alternating fashion; each solver can be designed to take advantage of the structure inherent in each subsystem. The simultaneous approach is generally thought to be more difficult and expensive to solve and hence the alternating method (or a variant thereof) is typically used in two or more spatial dimensions. As we will see below, the alternating approach fits well with the stochastic domain decomposition approach we describe to parallelize our computations.

The general QL approach has shown great promise in recent years, solving problems in meteorology [7], relativistic magnetohydrodynamics [14], combustion and convection in a porous medium [9], groundwater flow and transport of nonaqueous phase liquids [16], Stefan problems [4], semiconductor devices [30], and viscoelastic flows [31], phase change problems [3], multiphase flows [26], and low speed viscous flow [18], to name just a few. A thorough overview of PDE based moving mesh methods may be found in [15].

Recently, motivated by the alternating solution method, one of the authors has studied the parallel solution of the nonlinear MMPDE alone using a Schwarz based domain decomposition approach. In [12], Haynes and Gander propose and analyze classical, optimal and optimized Schwarz methods in one spatial dimension at the continuous level. A numerical study of classical and optimized Schwarz domain decomposition for 2D nonlinear mesh generation has been presented in [13]. In [10], a *monolithic* domain decomposition method, simultaneously solving a linear mesh generator coupled to the physical PDE, was presented for a shape optimization problem. The authors used an overlapping domain decomposition approach to solve the coupled problem.

In this paper, we present an efficient, parallel strategy for the solution of the moving mesh PDE based on a stochastic domain decomposition method proposed by Acebrón *et al.* [1]. The motivation is two–fold. First, we wish to reduce (by parallelization) the potential burden of having to solve an additional (mesh) PDE. Second, it is often mesh