J. Math. Study
doi: 10.4208/jms.v48n2.15.03

Vol. **48**, No. 2, pp. 125-167
June 2015

R-Adaptive Reconnection-based Arbitrary Lagrangian Eulerian Method-R-ReALE

Wurigen Bo[1], Mikhail Shashkov[2,*]

[1] *Computational and Computer Science Division, CCS-2, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*
[2] *X-Computational Physics, XCP-4, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*

Received 9 January, 2015; Accepted (in revised version) 22 May, 2015

Abstract. We present a new R-adaptive Arbitrary Lagrangian Eulerian (ALE) method, based on the reconnection-based ALE - ReALE methodology [5,41,42]. The main elements in a standard ReALE method are: an explicit Lagrangian phase on an arbitrary polygonal (in 2D) mesh, followed by a rezoning phase in which a new grid is defined, and a remapping phase in which the Lagrangian solution is transferred onto the new grid. The rezoned mesh is smoothed by using one or several steps toward centroidal Voronoi tessellation, but it is not adapted to the solution in any way. We present a new R-adaptive ReALE method (R-ReALE, where R stands for Relocation). The new method is based on the following design principles. First, a monitor function (or error indicator) based on Hessian of some flow parameter(s), is utilized. Second, the new algorithm uses the equidistribution principle with respect to the monitor function as criterion for defining an adaptive mesh. Third, centroidal Voronoi tessellation is used for the construction of the adaptive mesh. Fourth, we modify the raw monitor function (scale it to avoid extremely small and large cells and smooth it to create a smooth mesh), in order to utilize theoretical results related to centroidal Voronoi tessellation. In the R-ReALE method, the number of mesh cells is chosen at the beginning of the calculation and does not change with time, but the mesh is adapted according to the modified monitor function during the rezone stage at each time step. We present all details required for implementation of the new adaptive R-ReALE method and demonstrate its performance relative to standard ReALE method on a series of numerical examples.

AMS subject classifications: 65M08, 65M55,76N99
Key words: Gas Dynamics, R-adaptation, Reconnection, ALE.

1 Background and rationale

As in most standard Arbitrary-Lagrangian-Eulerian (ALE) methods [28], the main elements in a ReALE [42] simulation are an explicit Lagrangian phase in which the solution

---

*Corresponding author. *Email addresses:* `bowurigen@gmail.com` (W. Bo), `shashkov@lanl.gov` (M. Shashkov)

and grid are updated (without changing its connectivity), a rezoning phase in which a new grid is defined, and a remapping phase in which the Lagrangian solution is transferred onto the new rezoned grid. The ReALE method described in [42] differs from standard ALE method in one element only - it allows connectivity changes during the rezone stage. The rezone phase of ReALE includes both mesh movement and the reconnection procedure, which is done using the machinery of Voronoi diagrams, [1]. The ReALE rezone strategy consists of a special movement of generators. It is similar to Lagrangian motion in some sense, but also include a smoothing procedure based on the notion of centroidal Voronoi diagrams [16]. By construction, a Voronoi mesh is a valid mesh and therefore each Lagrangian step starts with a valid mesh. The main objective of [42] was to develop a robust reconnection-based ALE method in which averaged cell movement is close to Lagrangian. The ReALE method allows running complex simulations to completion without user intervention, while maintaining reasonable accuracy. In [42] we also presented a comparison of Lagrangian methods with ReALE on problems for which pure Lagrangian methods can run without mesh tangling. Examples of ReALE simulations can be found in [5,27,41,42].

The ReALE methods have significant potential with respect to adaptivity. First of all, repositioning (relocation or R-adaptivity) of the generators during the rezone stage can be related to some error indicator. Secondly, the number of generators (which defines the number of cells) can change with time to refine the mesh where it is needed (h-adaptivity). However, in [42] there was no attempt to explore adaptivity in framework of ReALE methods. In this paper we explore R-adaptivity in the framework of ReALE.

The need for adaptive methods is well recognized and there are numerous papers related to adaptation, see for example Chapter 14 in [40] and [30] and corresponding references herein, or the website `http://lsec.cc.ac.cn/~ttang/MMref`.

According to [40], any adaptive method is composed of three main ingredients: an error estimator or error indicator, an optimal-mesh criterion, and an algorithm of the strategy for mesh improvement. These ingredients answer the following questions: Where are mesh changes required? How should the optimal mesh be defined? How should the improved mesh be constructed?

Our adaptive ReALE methods are based on following well known basic design principles.[†]

The first design principle is to use a monitor (error indicator) function based on the Hessian of some flow parameter(s), which is a measure of interpolation error, [30,40]. In general, a monitor function $\phi(\mathbf{x},t) > 0$ is some measure or indicator of the error. In an ideal case its construction is based on error estimates. However, in reality, especially for non-linear hyperbolic problems, practitioners use much simpler and readily computable indicators of errors. In our case we choose to use a monitor function based on estimates

---

[†] Let us note that to describe the main ideas we intentionally use a loose style of presentation to avoid lengthy definitions and explanations. In the main text of the article we give strict definition of all notions and notations that we use.