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A LOCAL MULTILEVEL PRECONDITIONER FOR THE ADAPTIVE MORTAR FINITE ELEMENT METHOD*

Feng Wang Jinru Chen
Jiangsu Key Laboratory for NSLSCS, School of Mathematical Sciences, Nanjing Normal University, Nanjing 210023, China
Email: fwang@njnu.edu.cn jrchen@njnu.edu.cn

Abstract

In this paper, we propose a local multilevel preconditioner for the mortar finite element approximations of the elliptic problems. With some mesh assumptions on the interface, we prove that the condition number of the preconditioned systems is independent of the large jump of the coefficients but depends on the mesh levels around the cross points. Some numerical experiments are presented to confirm our theoretical results.

Mathematics subject classification: 65N30, 65N55, 65F10.
Key words: Mortar, Adaptive finite element method, Local multilevel method, Additive Schwarz method.

1. Introduction

In this paper, we present a local multilevel preconditioner for the adaptive mortar finite element method for the following second order elliptic problems:

$$\begin{cases} -\nabla\cdot(\rho\nabla u) = f, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\rho > 0$ is a piecewise constant, $f \in L^2(\Omega)$, and Ω is a polygonal domain.

The mortar finite element method is a technique for dealing with different discretization schemes on different subdomains [1, 2]. It is effective for solving problems with complicated geometries, heterogeneous material, multi-physics, and so on. In this paper, we use the mortar finite element method to handle the nonmatching meshes. Based on a posteriori error estimators, the adaptive finite element methods are now widely used to achieve better accuracy with minimum degrees of freedom. Combining the mortar approach and the adaptive finite element methods, many researchers propose different a posteriori error estimators (see [7, 8] and the references therein for details). The first author and his collaborator [23] also proposed some residual-based a posteriori error estimators, and the analysis does not require saturation assumptions or mesh restrictions on the interface which are often needed in the literature. However, there are rather few results on developing efficient solvers for the discrete problems. Based on quasi-uniform grids, Wohlmuth [26] and Gopalakrishnan [17] introduced V-cycle and W-cycle multigrid methods for the mortar finite element method for elliptic problems respectively. Xu and Chen [31] discussed a W-cycle multigrid algorithm for the mortar element method for the P_1 nonconforming element.

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Since the mesh is refined locally in the process of adaptivity, traditional multigrid methods, in which the smoothing is performed on all nodes, may not be optimal or quasi-optimal (see [19]). Wu and Chen [27] first proved that the local multigrid method, in which the smoothing was performed on new nodes and their "immediate" neighbors of each level, was optimal for the adaptive finite element method for the Poisson equation in two dimension. In [13, 15], Xu, etc., introduced and analyzed some local multigrid methods based on reconstructed adaptive grid, which was applied to the adaptive finite element methods for the elliptic problems with discontinuous coefficients [14]. Based on the adaptive grid, Xu and Chen [11, 12, 30] also proposed and analyzed some local multilevel methods for P_1 conforming and nonconforming element methods for the elliptic problems. Recently, Lu, Shi and Xu [18] considered the local multilevel methods for discontinuous Galerkin finite element method on adaptively refined meshes.

The purpose of this paper is to present a local multilevel preconditioner for the mortar finite element method for the second order elliptic problems with discontinuous coefficients. Since the finite element spaces are nonnested, intergrid transfer operators, which are stable under the weighted L^2 norm and energy norm, are introduced to exchange information between different meshes. On each level, the smoothing is performed on the new free nodes and the old free nodes associated with which the basis functions are changed. In addition, we also need to smooth on all the mortar side nodes on the finest level. With the assumption that each mortar side edge is the union of some whole nonmortar side edges (see Fig. 2.2 for an illustration), we prove that the condition number of preconditioned system is independent of the large jump of the coefficients but relies logarithmically on the mesh size around the cross points.

The remainder of the paper is organized as follows. In Section 2, we present the discrete problem and some notations. The local multilevel preconditioner is proposed in Section 3. In Section 4, we give the condition number estimate of the preconditioned system. Finally, we present some numerical experiments to confirm our theoretical results.

For convenience of discussions, we usually use inequalities $a \lesssim b$, $a \simeq b$ to replace $a \leq Cb$ and $cb \leq a \leq Cb$ with some multiplicative mesh size and coefficient independent constants $c, C > 0$ that depend only on the domain Ω and the shape (e.g., through the aspect ratio) of elements.

2. Preliminary

The weak form of the problem (1.1) is to find $u \in H_0^1(\Omega)$ satisfying

$$a(u, v) \triangleq (\rho \nabla u, \nabla v) = (f, v), \quad \forall v \in H_0^1(\Omega). \tag{2.1}$$

Let Ω be partitioned into non-overlapping polygonal subdomains $\{\Omega_i\}_{i=1}^N$. We only consider the geometrically conforming case, i.e., the intersection between the closure of two different subdomains is empty, a vertex, or an edge. The coefficient ρ is a constant when restricted to each subdomain Ω_i. We use Γ_{ij} to denote the common open edge of Ω_i and Ω_j, $\Gamma = \bigcup_{ij} \Gamma_{ij}$. Given an initial shape regular triangulation $\mathcal{T}_1(\Omega)$, which is conforming in each subdomain, $\{\mathcal{T}_l(\Omega),\ 2 \leq l \leq L\}$ is a set of triangulations generated by the adaptive finite element procedure [23]. We note that the resulting triangulation $\mathcal{T}_l(\Omega)$ can be non-matched across adjacent subdomain interfaces, so each Γ_{ij} can be regarded as two sides corresponding to the two subdomains Ω_i and Ω_j. We call one of the sides of Γ_{ij} as the mortar side and the other one as the nonmortar side. For each interface, we choose the side of the subdomain on which the coefficient is larger