

## Anisotropy of the Spectral Structures in Compressible Homogeneous Turbulent Shear Flow

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**Abstract.** The energy spectrum and energy transfer in compressible homogeneous turbulent shear flow are numerically investigated via high-accuracy direct numerical simulation. The Helmholtz decomposition method is employed to decompose the velocity field into a solenoidal component and a compressive one. It is found that the spectra of different velocity modes are strongly anisotropic over all the resolved scales and the specific properties of small-scale anisotropy are significantly influenced by the flow compressibility. The anisotropy of energy transfer process comes from an additional kinetic energy production term which acts as a dominant source at relatively large scales in the streamwise direction. After the redistribution of turbulent kinetic energy caused by the pressure-dilatation correlation in different directions, the energy fluxes due to advection are also expected to be anisotropic. The streamwise energy flux is predominantly large and passes down the kinetic energy to smaller scales, whereas the cross-streamwise and spanwise energy fluxes are less significant and pronounce an inverse energy transfer at relatively large scales.

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## 1 Introduction

Homogeneous turbulent shear flow (HTSF) refers to the turbulent flow which is generated and sustained by a simple mean flow of uniform shear rate. This flow regime invo-

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ives plenty of anisotropic physical mechanisms due to the direction preference of the preset uniform shear, while it preserves the simplicity of homogeneity. This particular flow configuration can be used to study the effects of shear in the absence of boundary effects and to bridge the gap between the idealized homogeneous isotropic turbulence (HIT) and the realistic flows with solid boundaries.

Since the seminal works by Rose [1] and Champagne et al. [2], HTSF has been widely investigated in various aspects both experimentally [3–8] and numerically [9–24]. Much attention has been placed on the small-scale anisotropy of HTSF, which has been studied by examining the Reynolds stress, distributions of vorticity and velocity derivatives, vorticity correlation tensors and velocity gradient moments, etc. [5, 6, 10, 11, 14, 15, 17–19, 22, 25–29]. These studies suggest the violation of the local isotropy hypothesized by Kolmogorov [30]. The universal small-scale isotropic statistics will be destructed when a mean velocity gradient exists and the resultant anisotropy will remain even at very high Reynolds number. The asymmetric distribution of spanwise vorticity component  $\omega_3$  [14, 15, 19, 22] and the anisotropic high-order statistics of the transverse derivative of streamwise velocity [5, 6, 25, 28, 29] are generally considered as the evidence of the small-scale anisotropy.

Due to the uniform shear, some general physical processes also behave anisotropically in HTSF. For example, the energy transfer process is severely affected by the turbulent kinetic energy production, which exists only in a selected direction. The physical mechanism of energy transfer has been extensively investigated in HIT by using both the spectral analysis method in spectral space and the filtering approach in physical space. In incompressible HIT, it is argued that the kinetic energy is transferred between different scales due to the nonlinear triad interactions [31–34]. The scale-to-scale energy transfer processes due to the advection, pressure, dilatation and dissipation effects for compressible HIT and Richtmyer-Meshkov instability induced mixing flow have also been discussed by analyzing corresponding spectral energy fluxes [35–38]. Recently, the low-pass filtering approach is utilized to calculate the local energy flux [37–44], which can quantify the transfer of kinetic energy at fixed points, shedding light on the structures of energy fluxes in physical space. In our study, we will show that the small-scale anisotropy can be depicted more clearly in spectral space through decomposition of different scales, which can be further used to clarify the detailed properties of the anisotropic flow structures.

It should be mentioned that most of the previous studies of HTSF mainly focus on incompressible situations. Only few compressible studies have conducted and are mostly limited to low-Mach number cases due to the employed Fourier pseudo-spectral method. In order to get an insight into the anisotropic properties of highly compressible HTSF, direct numerical simulations are carried out at turbulent Mach number up to 1 using a high-order hybrid numerical scheme [45]. The anisotropy of energy spectra and inter-scale energy transfer is analyzed in spectral space. The rest content of the paper is organized as follows. The governing equations and numerical methods are briefly introduced in Section 2. Then, the results about the properties of energy spectra and energy transfer are presented in Section 3. Finally, the concluding remarks are given in Section 4.