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Explicit Form of Simplified Grad's 13 Moments Distribution Function-Based Moment Gas Kinetic Solver with Unstructured Meshes for the Multiscale Rarefied Flow

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Abstract. It is essential to efficiently solve multiscale flows covering the continuum regime to the rarefied regime. The explicit form of Grad's 13-moment distribution function-based moment gas kinetic solver (G13-MGKS) has been proposed in our previous work [Comput. Math. Appl., 137 (2023), pp. 112–125], which demonstrates the potential for efficiently simulating continuum flows accurately and presenting reasonable predictions for rarefied flows at moderate Knudsen numbers on structured meshes. To further extend the solver's applicability to unstructured meshes, we propose the simplified version of Grad's 13 moments distribution function-based moment gas kinetic solver (SG13-MGKS) with an explicit form of the numerical flux in the present paper. The Shakhov collision model has been adopted and validated within the framework of SG13-MGKS to ensure the correct Prandtl number in the simulation. Additionally, a simplified treatment for the numerical fluxes has been adopted to minimize the need for complex calculations of the gradient of integral coefficients. The performance of SG13-MGKS has been evaluated in numerical cases of Couette flow with temperature differences, flow passing through the airfoil, and pressure-driven flow in a variable-diameter circular pipe. Our results demonstrate that SG13-MGKS can achieve reasonably accurate computational results at Knudsen numbers below 0.2. Benefiting from the avoidance of discretization in velocity space, G13-MGKS is able to be two orders of magnitude faster compared to the conventional discrete velocity method. Furthermore, the simplified form of numerical fluxes and the fewer gradients of integration coefficients enable the performance of SG13-MGKS on unstructured grids with a saving of about 4 times the computation time and 3 times the memory cost compared to the previous version of G13-MGKS.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07

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1 Introduction

Rarefied gas flows are encountered across diverse industrial scenarios, including the aerospace vehicle [1–3], photolithography industry [4, 5] and nuclear fusion devices [6, 7]. Conventional numerical simulations based on the Navier-Stokes (NS) equations, grounded in the continuum hypothesis, face limitations in accurately capturing flow phenomena under enhanced rarefaction effects [8,9]. Consequently, researchers are exploring the more fundamental Boltzmann equation to bridge this gap. In contrast to the conventional Computational Fluid Dynamics (CFD) solver relying on macroscopic variables, the Boltzmann equation adopts a statistical mechanics perspective [10], modeling gas behavior through the velocity distribution function of gas molecules, enabling a multiscale description from rarefied to continuum regimes [11].

Over the past few decades, significant progress has been made in numerical methods for solving the Boltzmann equation [12,13,40]. Representative methods fall into two main categories: stochastic methods and deterministic methods [32]. Among many stochastic methods, the Direct Simulation Monte Carlo (DSMC) [14] has achieved considerable success in studying high-speed rarefied flows. DSMC employs virtual gas molecule ensembles to simulate the collisions and transportation in molecular velocity space. Despite its nature of Lagrangian description, DSMC is challenged by the statistical noise significantly, particularly impacted in the low-speed flow simulations. In contrast, deterministic methods discretize the molecular velocity space by the discrete mesh, operating noise-free simulations based on the Eulerian perspective. As the most prominent deterministic approach, the Discrete velocity method (DVM) [15, 16] performs well in low-speed flow simulations. In recent years, multiscale numerical methods based on discrete velocity methods including the unified gas-kinetic scheme (UGKS) [38], discrete unified gas kinetic scheme (DUGKS) [33], improved discrete velocity method (IDVM) [17] and general synthetic iterative scheme (GSIS) [34, 41] have been widely developed. However, the additional velocity space discretization substantially increases computational demands. Tests indicate that under the same physical mesh, DVM requires significantly more computations compared to the conventional CFD solver, hindering its practical applicability.

To address rarefied effects beyond continuum assumptions and avoid extra discretization of the velocity space, Grad [18, 30] proposed a set of truncated velocity distribution functions. These truncated distribution functions express the unknown distribution as the polynomial form of moment variables such as density, velocity, temperature, stress, and heat flux. Building upon these moments of the distribution function, Grad derived the moment method, showing promising efficiency in moderately rarefied flows compared to the conventional CFD method. Subsequently, Struchtrup et al. [19] and Gu et al. [20] proposed the regularized approach to involve the constitutive models in the