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A Moving Pseudo-Boundary MFS for Three-Dimensional Void Detection

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Dedicated to Graeme Fairweather on the occasion of his 70th birthday. Mentor and friend for many years.

Abstract. We propose a new moving pseudo-boundary method of fundamental solutions (MFS) for the determination of the boundary of a three-dimensional void (rigid inclusion or cavity) within a conducting homogeneous host medium from overdetermined Cauchy data on the accessible exterior boundary. The algorithm for imaging the interior of the medium also makes use of radial spherical parametrization of the unknown star-shaped void and its centre in three dimensions. We also include the contraction and dilation factors in selecting the fictitious surfaces where the MFS sources are to be positioned in the set of unknowns in the resulting regularized nonlinear least-squares minimization. The feasibility of this new method is illustrated in several numerical examples.

AMS subject classifications: 65N35, 65N21, 65N38

Key words: Void detection, inverse problem, method of fundamental solutions.

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1 Introduction

An important subclass of inverse problems very often encountered in real life applications is represented by so-called inverse geometric problems. In such problems, the governing equation, material properties, boundary conditions and a portion of the geometry

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that is accessible are all known, whilst the portion of the geometry that is hidden from view is to be characterized with the help of an over-specified (Cauchy) condition on the exposed accessible surface. More precisely, in this paper we refer to applications in the nondestructive evaluation of three-dimensional voids (rigid inclusions and cavities) associated with isotropic steady-state heat conduction (i.e., Laplace's equation), for which both the boundary temperature and the heat flux are measured on the accessible exposed surface, the boundary condition on the void surface is known, while the geometry of the void is to be determined. This situation also occurs in electrical impedance tomography (EIT) where the boundary temperature and heat flux are replaced by the voltage and current flux, respectively. This inverse geometric problem has been solved by a variety of mesh dependent numerical methods, such as the finite difference (FDM), finite element (FEM) and boundary element (BEM) methods, see e.g., [7,8,15,16,23–25], and its numerical solution is, arguably, the most computationally intensive among problems belonging to the general class of inverse problems. Its inherent nature requires a complete regeneration of the mesh as the geometry evolves regardless of whether a numerical or analytical approach is employed to solve the associated direct problem. Consequently, the solution of inverse geometric problems via these traditional numerical methods is seriously dependent on the quality of the mesh since a simple topological mesh would cause the solution to be distorted and fails to complete the inverse calculation. Moreover, obtaining a high-quality mesh requires tedious (re)meshing in the inverse iterative process. Therefore, it is of crucial importance to find a proper and efficient numerical method to solve the inverse geometric problem under investigation in a stable and accurate manner.

The method of fundamental solutions (MFS) was first introduced as a numerical technique for direct problems in the late seventies in a paper by Mathon and Johnston [26] followed by applications to potential problems in papers by Fairweather and Johnston [10, 17]. In recent years, it has been used extensively for the numerical solution of various types of inverse problems, mainly because of the ease with which it can be implemented, see the recent survey papers [21, 22]. There are two MFS approaches related to the location of the MFS singularities (one of the most important issues concerning this meshless method). In the *static approach*, the singularities are pre-assigned and kept fixed throughout the solution process, whilst in the *dynamic approach*, the singularities and the unknown MFS coefficients are determined simultaneously during the solution process [9]. Because the coordinates of the singularities appear non-linearly, this approach leads to a non-linear least squares minimization problem. The obvious criticism of this approach is that in the case of linear boundary value problems one is required to solve a non-linear discrete problem at a high cost. On the other hand, the optimal placement of the singularities in the static approach is a major issue, see e.g., [2].

The detection of two-dimensional cavities and inclusions was investigated using a regularized MFS and the static approach in [3, 4, 18, 19]. Recently, the authors have proposed a dynamic moving pseudo-boundary MFS for void detection in two dimensions [20]. The purpose of this study is to extend and numerically implement this method to three-dimensional void detection problems. The paper is organized as follows. In Sec-