Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **16**, No. 3, pp. 771-804

DOI: 10.4208/aamm.OA-2023-0041
June 2024

Development of a Novel Nonlinear Dynamic Cavitation Model and Its Numerical Validations

Haidong Yu[1,2], Xiaobo Quan[3], Haipeng Wei[1], Matevž Dular[4] and Song Fu[2,*]

[1] *Beijing Institute of Astronautical System Engineering, Beijing 100076, China*
[2] *School of Aerospace Engineering, Tsinghua University, Beijing 100084, China*
[3] *China Academy of Launch Vehicle Technology, Beijing 100076, China*
[4] *Laboratory for Water and Turbine Machines, University of Ljubljana, Aškerčeva 6, 1000 Ljubljana, Slovenia*

Received 5 February 2023; Accepted (in revised version) 25 June 2023

Abstract. Aiming at modeling the cavitation bubble cluster, we propose a novel nonlinear dynamic cavitation model (NDCM) considering the second derivative term in Rayleigh-Plesset equation through strict mathematical derivation. There are two improvements of the new model: i) the empirical coefficients are eliminated by introduction of the nonuniform potential functions of ψ_v and ψ_c for growth and collapse processes respectively, and ii) only two model parameters are required, which both base on physical quantities–the Blake critical radius R_b and the average maximum growth radius R_m. The corresponding cavitation solver was developed by using OpenFOAM in which we implemented the modified momentum interpolation (MMI) method to ensure that the calculated results are independent of time step size. Three validation cases, namely numerical bubble cluster collapse, ultrasonic horn experiment, and hydrodynamic cavitation around slender body are employed. The results indicate that ψ_v and ψ_c can reveal the nonlinear characteristics for cavity accurately, and R_b and R_m can reflect the relevance between cavitation model and actual physical quantities. Moreover, it is discussed the potentiality of NDCM that is generally applied on the cavitating flow possessing with dispersed bubbly cloud.

AMS subject classifications: 76B10, 76M12

Key words: Cavitation model, Rayleigh-Plesset equation, bubble cluster collapse, OpenFOAM.

1 Introduction

Cavitation often occurs in liquid flows when the ambient pressure drops below a certain threshold. The cavitating bubbles will emerge gradually from "cracked" liquid medium

*Corresponding author.
Emails: fs-dem@tsinghua.edu.cn (S. Fu), yhd16@tsinghua.org.cn (H. Yu)

at its weak points [1]. Individual bubbles cluster and form a complex two-phase mixture cloud, which shape depends strongly on the structure of the flow field. The cavitation bubble cluster exhibits many unique characteristics, such as strong collapse accompanied by a shockwave, or the natural frequency far lower than single bubble's etc. [2, 3] There are many researches on single bubble dynamics which can be described by Rayleigh-Plesset type equation [4, 5]. However, an approach to build cavitation model based on Rayleigh-Plesset equation to investigate the dynamics of bubble cluster can be considered questionable.

Numerical simulation of cavitating flows and specifically the development of transport equation model (TEM) has received enormous attention from investigators in recently years. Instead of potential flow theory implemented in early engineering applications, the Eulerian's one field formulation (OFM) of the two-phase Navier-Stokes equation [6–9], which combines the properties of each phase as a single mixed one, is popularly applied as the methodology of multiphase model. The cavitation model for bubble cluster is embedded into the convective phase equation as source terms. There is also an alternative approach based on Eulerian-Lagrangian method [10–12], which can depict dynamic characteristic of each individual bubble respectively, but is not within the present framework.

The prototype of TEM was introduced by Kubota [13] who assumed that the bubble nuclei are uniformly distributed in the flow, and the simplified Rayleigh-Plesset equation, which considered the SGS bubble interaction was used to determine the change of radius and consequently the mixture density in each computational cell. The advantage of this approach is that the dynamic response of local equilibrium bubbles can be estimated precisely. However, solving the nonlinear transport equation often faces the difficulty of convergence and its application was merely limited in studying steady flow.

In order to develop the Kubota's method, the modeling strategy was focused on larger scale of bubble cloud rather than the tiny scale of individual bubbles. A different approach with the same form of transport equation models had already been proposed in literature [6–9], in which the nonlinear ODE is replaced by a convective equation of void fraction. The mass transfer rate of bubbles is contained implicitly into source terms. It has the advantage that the convective character of equation is more appropriate for describing the topological evolution of bubble cluster particularly in unsteady situations. In the literature, there are two main approaches to build the empirical phase-transition laws associating with surrounding pressure. Merkel et al. [14–16] proposed simplest formulation based on dimensional analysis that defined the characteristic velocity u_∞ or temporal scale t_∞. However, these parameters are prescribed as a constant during the whole dynamic period, especially in collapsing process, which would cause large deviation. In order to make the model more physical, the void fraction was regarded equivalently as a group of identical bubbles. Schnerr et al. [17–20] simplified the Rayleigh-Plesset equation that only the linear part was used to redefine the velocity scale as $\sqrt{2|p-p_{sat}|/3\rho_l}$ for both growth and collapse situations. The advantage of this formulation is that it follows, to some part, a physical law and reduces the complexity. Nevertheless, the empirical