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An Efficient Method for the RCS Reduction of Cavities in the Ground Plane

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Abstract. This paper investigates the reduction of backscatter radar cross section (RCS) for cavities embedded in the ground plane. It is established by placing a multilayered radar absorbing material (RAM) at the bottom of the cavities. The minimization of the backscatter RCS is formulated as a bound constrained optimization problem. A new vertical mode expansion method is developed to solve the scattering problem, and it is integrated into the gradient projection method to obtain the optimal composite coating. Numerical examples show that the method we developed is efficient and the backscatter RCS can be significantly reduced.

AMS subject classifications: 35Q60, 35J05, 49M37

Key words: Vertical mode expansion, RCS reduction, optimal design, gradient projection.

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1 Introduction

The scattering of cavities in the infinite ground plane is of great importance for its industrial application. There has been a considerable interest in computation and design of cavities both in the engineering community and the mathematical community. Radar cross section (RCS) is an important measure for the detection of a target by radar system. Reducing the RCS from cavities has been an important problem in wave propagation with many practical applications. One of the effective approaches for RCS reduction is to use multilayered radar absorbing materials (RAM) to absorb the energy. In this paper, we consider two dimensional cavities which are embedded in the ground plane and illuminated by a time harmonic plane wave. A multilayered RAM is coated horizontally at the bottom of the cavities for the reduction of backscatter RCS, as shown in Fig. 1.

The scattering problem by cavities has been extensively studied in terms of both mathematical analysis and numerical simulation. The mathematical analysis for the well-posedness of the variational problems can be found in [1–3], where the existence and

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uniqueness for the scattering problem of a cavity were established in a rather general setting. The stability estimates with explicit dependence on the wavenumber were obtained in [8, 9]. The overfilled cavity problems, where the filling materials inside the cavity may protrude into the space above the ground plane, were investigated in [12, 15, 19, 29]. The scattering from cavity embedded in an impedance ground plane was examined in [11,13]. A variety of numerical methods have been developed for solving this problem. Standard methods include finite difference [7,32], finite element [17,30], boundary element [14,31] and hybrid method [16,27]. For large cavities or cavities coated with a multilayered RAM, the calculation of the field becomes difficult for the usual numerical methods, since small grid or mesh size must be taken to resolve the highly oscillatory fields and the very thin coated layers inside the cavities. Mode matching method is an alternative approach for scattering problems involving large cavities, however, the modal approach is limited only to cavities with uniform cross sections [4, 10, 20, 23]. We refer the reader to the survey [18] and the references cited therein for a comprehensive review on the analysis and computation of the cavity scattering problem.

Many optimization techniques have been proposed to reduce the RCS of a target using multilayered RAM [24, 26]. The genetic algorithm (GA) is an efficient optimizer to obtain the best combination of the RAM among available discrete database of materials. However, when the admissible set for the material is not discrete, it is computationally unaffordable to employ GA directly to solve the problem. In [5, 6], the gradient-based sequential quadratic programming method is addressed for the optimal design problem, where an efficient adjoint state method is introduced to find the gradient of the objective funtion. However, the gradient based method generally leads to a local minimum, a good initial guess is essential to obtain the satisfactory results.

In this paper, we develop a vertical mode expansion method (VMEM) to solve the cavity scattering problem. The structure of the cavities can be separated into a number of regions, where each region is piecewise uniform along the vertical direction. In each uniform region, using a proper reference solution, the governing equation becomes separable and the wave field is expanded in eigenmodes of a 1D differential operator. The expansion coefficients are solved from a linear system obtained by matching the wave field at the interfaces between neighboring regions. The method has the advantage of avoiding discretizing one spatial variable, only the coefficient of the each mode is solved. Comparing with the mode matching methods in [4, 10], VMEM is applicable to multiple cavity scattering problems directly, and it has the flexibility to be extended to trapezoidal cavities, i.e., the bottom of the cavity is a trapezoidal surface with rectangular corners. Moreover, the analysis of scattering of cavities embedded in the impedance ground plane can be unified in the same computational framework. It is extremely fast and efficient, which provides a powerful computation engine of the optimization problem.

Our aim in this paper is to design the optimal RAM coating, so that the RCS is reduced over a range of incident angles. The problem is formulated as a simple bound constrained optimization problem. A gradient projection method is integrated with VMEM to solve the problem. The basic idea is to approximate the problem as quadratic program-