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A Novel High-Order Shock-Capturing Limiter for RKDG Method on Unstructured Meshes via TENO Selection Strategies

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Abstract. There is a rapidly growing need for high-order numerical schemes on unstructured meshes in the numerical simulation of compressible flows. However, numerous barriers exist to achieve the desired high-order properties on unstructured meshes. The traditional finite-difference method is inadequate for complex unstructured meshes, whereas the finite-volume method introduces additional complexities, e.g., too large stencil size with too many neighboring cells for high-order reconstruction. The Runge–Kutta Discontinuous Galerkin (RKDG) method possesses inherent advantages for unstructured meshes, since it can ensure the high-order property regardless of the type of mesh within a compact stencil. On the other hand, the main weakness of the original RKDG method is their inability to capture discontinuities without artificial numerical oscillations. To tackle this issue, the troubled cell indicator and the slope limiter are integrated with the RKDG method to capture physical discontinuities, where the slope limiter modifies the numerical solutions of the troubled cells detected by the troubled cell indicator. In this work, we propose a new troubled cell indicator and a new limiter based on Targeted Essentially Non-Oscillatory (TENO) schemes for unstructured meshes. The new TENO troubled cell indicator can be applied to unstructured meshes with the compact property and ability to detect troubled cells. For each interface of the targeted troubled cells, the new TENO limiter is utilized to select a smooth numerical solution between the troubled cell and its immediate neighboring cell. Then, a novel weighting strategy is proposed to obtain the final reconstructed numerical solution from the candidate numerical solutions

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at each interface. It is worth noting that the conventional weighting strategy computes the final reconstructed numerical solution based on the area of corresponding neighboring cells, whereas the newly proposed weighting strategy determines the final reconstructed numerical solution regarding the smoothness of each candidate numerical solution from different interfaces rather than their areas. A set of benchmark cases, including strong shockwaves and a broad range of flow length scales, is simulated to illustrate the performance of the newly proposed limiter in comparison to the Weighted Essentially Non-Oscillatory (WENO) limiter on unstructured meshes. The newly proposed scheme significantly enhances the WENO limiter, exhibiting superior robustness and low-dissipation properties.

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Key words: PDEs, Runge-Kutta Discontinuous Galerkin method, TENO, WENO, Hyperbolic conservation laws.

1 Introduction

As the governing equation of compressible flows, the solution of the hyperbolic conservation law contains broadband flow scales and shocks, even if the initial condition is smooth. Traditionally, the finite-difference method and the finite-volume method are developed to solve the hyperbolic conservation law. Despite maturing over decades of development, the two methods still retain inherent drawbacks that remain unresolved. For example, the finite-difference method is incapable of handling unstructured meshes. Meanwhile, the finite-volume method encounters difficulties in achieving high-order accuracy solely relying on a compact stencil, although some research [1, 2] has been conducted on this issue. To address these limitations, the Discontinuous Galerkin (DG) method [3] is proposed as a class of the finite-element method, which solves the transport equation on triangular meshes for the first time. In addition to the aforementioned advantages, the DG method exhibits notable parallel efficiency and $h-p$ adaptivity properties.

Although the DG method offers the aforementioned benefits over traditional finite-difference and finite-volume methods, it encounters fundamental issues when solving equations with discontinuities. Specifically, the DG method, without additional modifications, is solely limited to computing the numerical solutions of the hyperbolic conservation law containing smooth regions and weak discontinuities. However, when it comes to strong shocks and contact discontinuities, spurious oscillations or even nonlinear instabilities may appear in the numerical solution of the DG method due to the Gibbs phenomenon [4]. To remedy this issue, nonlinear limiters are employed, modifying the so-called "troubled cell" with a less oscillatory polynomial, where troubled cells are mainly contained by discontinuities and high-wavenumber fluctuations. In the beginning, Cockburn et al. [5] utilized the TVB scheme as the nonlinear limiter. In spite of the fact that the TVB limiter has several drawbacks, such as its inability to maintain