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A Robust, Fully Adaptive Hybrid Level-Set/Front-Tracking Method for Two-Phase Flows with an Accurate Surface Tension Computation

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Abstract. We present a variable time step, fully adaptive in space, hybrid method for the accurate simulation of incompressible two-phase flows in the presence of surface tension in two dimensions. The method is based on the hybrid level set/front-tracking approach proposed in [H. D. Ceniceros and A. M. Roma, *J. Comput. Phys.*, 205, 391-400, 2005]. Geometric, interfacial quantities are computed from front-tracking via the immersed-boundary setting while the signed distance (level set) function, which is evaluated fast and to machine precision, is used as a fluid indicator. The surface tension force is obtained by employing the mixed Eulerian/Lagrangian representation introduced in [S. Shin, S. I. Abdel-Khalik, V. Daru and D. Juric, *J. Comput. Phys.*, 203, 493-516, 2005] whose success for greatly reducing parasitic currents has been demonstrated. The use of our accurate fluid indicator together with effective Lagrangian marker control enhance this parasitic current reduction by several orders of magnitude. To resolve accurately and efficiently sharp gradients and salient flow features we employ dynamic, adaptive mesh refinements. This spatial adaption is used in concert with a dynamic control of the distribution of the Lagrangian nodes along the fluid interface and a variable time step, linearly implicit time integration scheme. We present numerical examples designed to test the capabilities and performance of the proposed approach as well as three applications: the long-time evolution of a fluid interface undergoing Rayleigh-Taylor instability, an example of bubble ascending dynamics, and a drop impacting on a free interface whose dynamics we compare with both existing numerical and experimental data.

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1 Introduction

Multi-phase flows are the source of numerous nonlinear processes of both scientific and technological relevance. These flows are characterized by a complex motion of fluid interfaces that separate masses of fluids with different material properties and the free boundaries can undergo significant deformations and topological transitions.

Due to the multi-component nature of the flow, the fluid interfaces are subjected to surface tension and this interfacial force plays a fundamental role in nearly all multiphase flows of physical interest. Moreover, multiphase flows are typically multi-scale; the important phenomena of drop coalescence and break-up [1–3] as well as the generation of short capillary waves [4, 5] are just a few examples that exhibit the presence of multiple length scales. Therefore, an effective numerical method for the simulation of multiphase flows is required both to accurately represent the singularly supported interfacial forces and all the physically relevant flow quantities and to faithfully capture the disparate length scales. The method we propose here responds to these requirements in a computationally efficient and robust manner.

Numerical methods for computing multiphase flows can be broadly divided into two types: capturing and tracking. In capturing methods, such as the continuum surface force (CSF) model [6], the level set approach [7, 8], the phase field method [9–15], and the volume-of-fluid (VOF) method [16–19], the fluid interface is implicitly defined through a globally specified scalar function (the mass density, a signed distance function, an order parameter, or a volume fraction) which acts as a fluid indicator. These methods capture the interface motion on an Eulerian grid and handle automatically changes in interfacial topology. Front-tracking methods [20–25] on the other hand, use a separate grid to explicitly follow the interface motion and thus can achieve, in general, an accurate representation of geometric interfacial quantities.

In an attempt to overcome some of the inherent limitations of the aforementioned methodologies, there has been in recent years an increased attention to develop hybrid approaches [19, 26–31]. These hybrid strategies seek to exploit the best features of two different approaches by merging them into one method. The method presented here follows this philosophy. It originates from the hybrid level-set/front-tracking (LeFT) setting proposed in [27], from the fast and accurate fluid indicator developed in [30], and from the adaptive immersed boundary (IB) method first introduced by Roma, Peskin, and Berger [32] (for a recent, alternative adaptive version of the IB method see [33]). Moreover, the method combines these approaches with a mixed Lagrangian/Eulerian tension force representation proposed by Shin et al. [34].