Shallow Water Moment Models for Bedload Transport Problems

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Abstract. In this work a simple but accurate shallow model for bedload sediment transport is proposed. The model is based on applying the moment approach to the Shallow Water Exner model, making it possible to recover the vertical structure of the flow. This approach allows us to obtain a better approximation of the fluid velocity close to the bottom, which is the relevant velocity for the sediment transport. A general Shallow Water Exner moment model allowing for polynomial velocity profiles of arbitrary order is obtained. A regularization ensures hyperbolicity and easy computation of the eigenvalues. The system is solved by means of an adapted IFCP scheme proposed here. The improvement of this IFCP type scheme is based on the approximation of the eigenvalue associated to the sediment transport. Numerical tests are presented which deal with large and short time scales. The proposed model allows to obtain the vertical structure of the fluid, which results in a better description on the bedload transport of the sediment layer.

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1 Introduction

Sediment transport and the morphological evolution of riverbeds due to the deposition and erosion are an active topic in the study of fluvial processes. The sediment is transported by the river current as suspended load (finer fractions carried by the flow) and

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bedload (coarse fractions which move close to the bottom rolling, jumping and sliding), see [16].

The study of sediment transport focuses on understanding the relationship that exists between the movement of water and the movement of sedimentary materials. We are interested here in the so-called bedload transport, which is the type of transport that mainly happens near the bottom. In bedload, the sediment grains roll or slide along the bed. Single grains may even jump over the bed a length proportional to their diameter, losing for instants the contact with the soil, but mainly staying near the bed. A first approach to model bedload transport is to couple the Shallow Water equations with the so-called Exner equation (see [17]). This equation depends on the empirical definition of the solid transport flux for the bedload transport. Several formulations have been proposed, see for instance [25,39,46]. This approach has been extensively used to describe bedload transport, see [6,7,10,29,37,41], among many others.

The description of these empirical formulae for the solid transport discharge is usually based on the velocity of the fluid, which is given by the Shallow Water model describing the hydrodynamic component. Nevertheless, this velocity corresponds to the averaged value in the water column. One would expect to use the near bed velocity of the fluid for bedload transport (see [19] and references there in), however the lack of the vertical profile for the velocity due to the Shallow Water approach makes it impossible.

In recent years, effort has been made in using more complex shallow type models in order to have a better description of the fluid in the vertical direction. One possible direction is the use of the multilayer approach [2,21,40]. This approach allows us to recover the vertical profile by subdividing the domain along the normal direction in shallow layers, and applying the thin-layer hypothesis within each layer. Thus, the domain is discretized in the vertical direction, leading to a larger system whose number of equations and unknowns depends on N, where N is the number of layers. A drawback of this approach is that many layers should be employed if very complex profiles of velocity have to be recovered, leading to a high computational cost (although much lower than solving the full 3D Navier-Stokes system). In [3], an application to bedload transport problem is simulated by using a multilayer model and the Grass formula, which allows the authors to use the velocity near the bottom and not the depth-averaged velocity as in the Shallow Water model. It should be noted that the resulting multilayer model seems to be hyperbolic based on numerical simulations, although this question remains open for arbitrary numbers of layers. Moreover, no analytical explicit general expression for the eigenvalues is known.

An extended Shallow Water model was derived in [35]. The model uses a polynomial expansion of the horizontal velocity along the vertical axis such that complicated velocity profiles can be represented efficiently using an extended set of variables that includes the basis coefficients of the polynomial basis. This approach is called moment method and the resulting model is the Shallow Water Moment model (SWM). Even though the model showed good results for standard test cases, it was shown in [32] that the hyperbolicity