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The Effects of Spatial Reconstruction and Flux Solver on the Performance of High-Order Finite-Volume Compressible Flow Solvers

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Abstract. Most high-order computational fluid dynamics methods for compressible flows are based on the Riemann solver for the flux evaluation and high-order interpolation or reconstruction such as the Weighted Essential Non-Oscillatory (WENO) scheme for spatial accuracy. The advantage of this kind of combination is the easy implementation and the ability to achieve the required spatial accuracy. However, despite the extensive research on high-order spatial reconstruction in the past, solvers coupling high-order space and time schemes have not been systematically evaluated. In this paper, based on the same fifth-order finite volume method (FVM), comparisons of the performance of the same flux solver with different reconstructions and the same reconstruction but different flux solvers are carried out on a structured mesh. For reconstruction, the TENO scheme and classic WENO-Z reconstruction have been chosen as representative methods. Meanwhile, for the flux solver, Lax-Friedrichs (LF) Riemann solver, HLLC solver, and GKS are considered. Through a series of simulated comparison cases, the unique characteristics of GKS and TENO have been demonstrated. Overall, the comparisons suggest that proper spatial and temporal coupling is important for accurate shock and vortex capturing.

AMS subject classifications: 76N17, 76M12, 35Q30, 35Q20
Key words: TENO reconstruction, Gas-kinetic scheme (GKS), Lax-Friedrichs Riemann solver, high-order scheme.

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1 Introduction

The development of high-order schemes for compressible flows has achieved great success in resolving complex vortex structures and capturing flow discontinuities. Compared with the finite difference method, the high-order finite-volume method has greatly improved. The finite volume method comprises the three main substeps, namely, the spatial reconstruction, flux evaluation, and temporal discretization. Most of the efforts were focused on building a novel high-order reconstruction method, which indeed achieved great success. The successful high-order reconstruction methods include the essentially non-oscillatory (ENO) and weighted essentially non-oscillatory (WENO) scheme [11, 16, 21]. There exist many modified versions of WENO, such as WENO-JS [16], WENO-Z [3], WENO with adaptive order WENO-AO [1], multi-resolution WENO [38], and target ENO(TENO) [8], etc.

In addition to the reconstruction, the flux evaluation and temporal discretization method also play dominant roles in the overall performance of the scheme. Generally, in terms of the flux solver, the approximate Riemann solvers are commonly used, such as Roe [25], Advection Upstream Splitting Method (AUSM) [20], and Harten-Lax-van Leer contact (HLLC) [30]. Since the Riemann solvers with a forward-Euler step has only a first-order temporal accuracy, the Runge-Kutta (RK) method [12] is usually adopted to advance the solution in time, making the high-order schemes stable and accurate in time.

Different from the Riemann solvers, there are other flux solvers to treat the time-dependent interface fluxes, such as generalized Riemann problem (GRP) [19], Arbitrary accuracy DERivative (ADER) [26], the gas-kinetic scheme (GKS), etc. In this paper, we mainly focus on the GKS solver. During the past two decades, GKS has shown its ability to accurately recover the Euler and Navier-Stokes solutions [34, 36]. GKS is mainly based on the Bhatnagar–Gross–Krook (BGK) collision model [2] and by directly integrating the BGK Boltzmann equation along the trajectory line, a time-dependent gas distribution function can be obtained, which has the advantage of high order in time and space. By using the time-dependent gas distribution function at the cell interface, the cell interface flux can be updated. In the previous study, it was found that the advantages of high-order GKS (HGKS) are as follows: (1) GKS presents a gas flow evolution from the kinetic scale to hydrodynamic scale, which not only provides accurate solutions for the smooth regions of the flow field but also effectively captures shockwaves in the discontinuous regions; (2) the inviscous and viscous terms are obtained simultaneously from the gas distribution function containing both equilibrium and non-equilibrium flow properties; (3) the flux in GKS has multi-dimensional properties [36], with contributions from both normal and tangential derivatives of flow variables around a cell interface; (4) compared with the traditional time-space independent Riemann Solver, the multi-stage multi-derivative (MSMD) [15] methods such as the two-stage fourth-order scheme (S2O4) [24] can provide the same order time integration accuracy with fewer middle stages due to the considerations of the time-derivative of the interface flux in GKS.

In the previous work, Yang compared the performance of GKS and HLLC flux us-