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Empirical Bayes Test for the Parameter of Rayleigh Distribution with Error of Measurement*

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Abstract: For the data with error of measurement in historical samples, the empirical Bayes test rule for the parameter of Rayleigh distribution is constructed, and the asymptotically optimal property is obtained. It is shown that the convergence rate of the proposed EB test rule can be arbitrarily close to $O(n^{-\frac{1}{2}})$ under suitable conditions.

Key words: error of measurement, empirical Bayes, asymptotic optimality, convergence rate

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1 Introduction

Empirical Bayes (EB) approach has been studied extensively by the researchers, and the readers are referred to literature [1]–[8].

Data with error of measurement take place in many fields, including biology, ecology, geology and medicine (see [9]–[10]). Up to now, empirical Bayes test problem for the parameter of distribution with error of measurement has not been studied by any researcher. Rayleigh distribution plays an important role in reliability analysis. In this paper, we discuss the empirical Bayes test for the parameter of Rayleigh distribution with error data of measurement.

Let X have a conditional density function

$$f(x \mid \theta) = \frac{x}{\theta^2} \mathrm{e}^{-\frac{x^2}{2\theta^2}}, \tag{1.1}$$

where θ is an unknown parameter. Denote the sample space by $x \in \Omega = \{x \mid x > 0\}$ and

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parameter space by $\Theta = \{\theta \mid \theta > 0\}$. In this paper, we discuss the one-sided test problem

$$\mathrm{H}_0: \ \theta \le \theta_0 \Longleftrightarrow \mathrm{H}_1: \ \theta > \theta_0, \tag{1.2}$$

where θ_0 is a given positive constant.

To construct EB test function, we have firstly loss functions

$$L_0(\theta, d_0) = \begin{cases} 0, & \theta \le \theta_0; \\ a\Big[1 - \Big(\dfrac{\theta_0}{\theta}\Big)^2\Big], & \theta > \theta_0, \end{cases} \qquad L_1(\theta, d_1) = \begin{cases} a\Big[\Big(\dfrac{\theta_0}{\theta}\Big)^2 - 1\Big], & \theta \le \theta_0; \\ 0, & \theta > \theta_0, \end{cases}$$

where $a > 0$, $d = \{d_0, \ d_1\}$ is action space, d_0 and d_1 imply acceptance and rejection of H_0.

Assume that the prior distribution $G(\theta)$ of θ is unknown. Then we have the randomized decision function

$$\delta(x) = P(\text{accept } \mathrm{H}_0 \mid X = x). \tag{1.3}$$

And the risk function of $\delta(x)$ is shown by

$$\begin{aligned} R(\delta(x), G(\theta)) &= \int_\Theta \int_\Omega [L_0(\theta, d_0) f(x \mid \theta)\delta(x) + L_1(\theta, d_1) f(x \mid \theta)(1 - \delta(x))]\mathrm{d}x\mathrm{d}G(\theta) \\ &= a\int_\Omega \beta(x)\delta(x)\mathrm{d}x + C_G, \end{aligned} \tag{1.4}$$

where

$$C_G = \int_\Theta L_1(\theta, d_1)\mathrm{d}G(\theta), \qquad \beta(x) = \int_\Theta \Big[1 - \Big(\frac{\theta_0}{\theta}\Big)^2\Big] f(x \mid \theta)\mathrm{d}G(\theta). \tag{1.5}$$

The marginal density function of X is given by

$$f_G(x) = \int_\Theta f(x \mid \theta)\mathrm{d}G(\theta) = \int_\Theta \frac{x}{\theta^2}\mathrm{e}^{-\frac{x^2}{2\theta^2}}\mathrm{d}G(\theta).$$

By (1.5) and simple calculations, we have

$$\beta(x) = u(x) f_G(x) + v(x) f_G^{(1)}(x), \tag{1.6}$$

where $f_G^{(1)}(x)$ is the first order derivative of $f_G(x)$, and

$$u(x) = 1 - \frac{1}{4}\theta_0 x^{-2}, \qquad v(x) = \frac{1}{2}\theta_0 x^{-1}.$$

Using (1.4), we obtain the Bayes test function as follows:

$$\delta_G(x) = \begin{cases} 1, & \beta(x) \le 0; \\ 0, & \beta(x) > 0. \end{cases} \tag{1.7}$$

Further, we can get the minimum Bayes risk

$$R(G) = \inf_\delta R(\delta, G) = R(\delta_G, G) = a\int_\Omega \beta(x)\delta_G(x)\mathrm{d}x + C_G. \tag{1.8}$$

When the prior distribution of $G(\theta)$ is known and $\delta(x) = \delta_G(x)$, $R(G)$ can be obtained. However, when $G(\theta)$ is unknown, so that $\delta_G(x)$ cannot be made use of, we need to introduce EB method.

2 Construction of EB Test Function

Under the following assumptions, we are to construct the EB test function. Let (X_1, θ_1), (X_2, θ_2), $\cdots$, (X_n, θ_n) and $(X_{n+1}, \theta_{n+1}) \widehat{=} (X, \theta)$ be independent random vectors, where θ_i $(i = 1, \cdots, n)$ and θ are indepently identically distributed (i.i.d.) and have common prior distribution $G(\theta)$. Let X_1, X_2, $\cdots$, X_n, X be sequence of mutually independent random