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# ANALYSIS OF TRANSMISSION LINE MODEL WITH UNCERTAIN PARAMETER USING THE PC–FDTD METHOD

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**Abstract.** Voltage and current wave propagation on transmission lines is commonly described by the telegrapher's equations. However, in practical applications, the parameters of the telegrapher's equations – such as inductance and capacitance – often exhibit uncertainty due to manufacturing tolerances, material inhomogeneities, and environmental variations. This study presents a numerical framework for solving the telegrapher's equations with uncertain parameters using a combination of Polynomial Chaos Expansion and the Finite Difference Time Domain (FDTD) method. The proposed approach enables efficient uncertainty quantification while maintaining computational tractability. In addition to the numerical formulation, the conditional stability of our method is analyzed, and the discrete dispersion relation is derived and compared with the continuous dispersion relation. The results demonstrate that the PC–FDTD method is a robust and accurate tool for modeling wave behavior in transmission lines under parameter uncertainty.

**Key words.** Maxwell's equations, telegrapher's equations, FDTD, polynomial chaos, uncertainty.

## 1. Introduction

Maxwell's equations are a set of fundamental equations in electromagnetism that describe how electric and magnetic fields propagate and interact with matter. These equations not only form the theoretical foundation of classical electrodynamics but also underpin a vast array of modern technologies, from wireless communication to power transmission. One notable application of Maxwell's equations is their reduction to simpler forms under specific assumptions, such as the derivation of the telegrapher's equations, which model the voltage and current in an electrical transmission line.

The telegrapher's equations are a partial differential equations that account for the distributed resistance, inductance, capacitance, and conductance of transmission lines. They plays a critical role in electrical engineering and mathematical physics, particularly in analyzing signal propagation, dispersion, and attenuation in conductive media. Mathematically, they are expressed in both hyperbolic and parabolic forms, depending on the modeling context, and have been extensively studied using both analytic and numerical methods.

Recent research efforts have focused on advancing solution techniques for the telegrapher's equations. Analytical approaches, such as the Laplace and Fourier transform methods, have been utilized to derive exact solutions under specific initial and boundary conditions [7, 10]. However, due to the complexity and variability of practical systems, numerical methods are increasingly being used to approximate solutions for more general and complex scenarios. Frequency domain methods have fewer numerical issues and are easier to implement. However, time domain approaches have a significant advantage in being able to model a broad range of frequencies in a single simulation. Time domain numerical methods – such as finite difference methods, finite element methods – [12, 13], and in particular

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the finite-difference time-domain (FDTD) method, are the most common way of approximating the time-domain response of transmission lines [4].

Extensions to the telegrapher's equations include incorporating interactions with other components which are modeled by coupling to additional equations such as Maxwell's equations, circuit theory, and thin-wire models [14]. Memory effects are modeled by coupling to additional constitutive laws, or modifying the telegrapher's equations by adding fractional derivatives [3]. In dispersive electromagnetic models, fractional derivative mechanisms can be accurately represented by endowing dielectric parameters with probability distributions accounting for microscale variation [6]. This is one motivation for the current work of treating random model parameters.

In practical applications, uncertainty in the parameters of the telegrapher's equations, such as resistance or inductance per unit length, can arise due to manufacturing tolerances, environmental effects, or material inconsistencies. This uncertainty poses significant challenges in modeling and simulation, as it can affect the reliability and accuracy of the solution. Many recent studies have begun to incorporate stochastic methods and uncertainty quantification frameworks in computational electromagnetics [5, 11, 15]. To our knowledge, random parameters in telegrapher's equations has not yet been considered.

The objective of this study is to develop a numerical method using polynomial chaos expansion for the telegrapher's equations with uncertain parameters. By treating these parameters as random variables, we aim to propose a robust computational framework that can accommodate uncertainty and improve predictive accuracy. The methodology will be validated through comparative studies and numerical experiments.

## 2. Model Formulation

**2.1. Random Telegrapher's Equations.** For voltage and current waves on transmission lines, application of Kirchoff's voltage and current laws, which are contained in Maxwell's equations, leads to the telegrapher's equations in the time domain:

$$\frac{\partial V}{\partial x} + L\frac{\partial I}{\partial t} + RI = 0, \tag{1a}$$

$$\frac{\partial I}{\partial x} + C\frac{\partial V}{\partial t} + GV = 0, \tag{1b}$$

where $V(x,t)$ and $I(x,t)$ are respectively the line voltage and current, while $R, L, G,$ and $C$ are respectively the distributed resistance, inductance, conductance, and capacitance of the line, in units of Ω/m, H/m, S/m, and F/m, respectively, and $(x,t) \in [0,X] \times [0,T]$. The system is completed by initial and boundary conditions, for example homogeneous initial conditions and homogeneous Dirichlet boundary conditions. The system of equations can be combined to derive a wave equation with only one dependent variable, either in terms of the voltage $V$ or the current $I$:

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2} + (RC + LG)\frac{\partial V}{\partial t} + RGV,$$

or

$$\frac{\partial^2 I}{\partial x^2} = LC\frac{\partial^2 I}{\partial t^2} + (RC + LG)\frac{\partial I}{\partial t} + RGI.$$

If parameters $R$ and $G$ are zero, the equations model the lossless case.