East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.2025-051.060425

Vol. **16**, No. 3, pp. 552-572
August 2026

Painlevé Analysis and Analytic Solutions of a Variable-Coefficient Sawada-Kotera System in Shallow Water, Ion-Acoustic Waves and Fluid Flow Dynamics

Hao-Qing Chen, Guang-Mei Wei* and Yu-Xin Song

LMIB and School of Mathematics Sciences, Beihang University, Beijing 100083, China.

Received 13 March 2025; Accepted (in revised version) 6 April 2025.

Abstract. A variable-coefficient Sawada-Kotera system is investigated that models the nonlinear behaviors of waves in shallow water, ion-acoustic waves in plasma environments and fluid flow dynamics. The Painlevé integrability is tested by the WTC method with the simplified form of Krustal. The Hirota bilinear method is employed to derive the bilinear form. Consequently, we obtain a variety of analytic solutions, including soliton, lump, and breather solutions. In addition, the interactions between the lump soliton and one stripe soliton, among with the breather soliton and one stripe soliton are discussed.

AMS subject classifications: 35C08, 35G50, 47J35

Key words: Sawada-Kotera system, Painlevé integrability, soliton solution, lump solution, breather solution.

1. Introduction

Nonlinear evolution equations (NLEEs) are partial differential equations that involve time variable t and arise from a wide array of nonlinear models across different fields of mechanics, physics, and engineering. These equations are of great importance due to their capability to describe complex dynamic systems and nonlinear phenomena. Therefore, they attracted widespread attention in the fields of theoretical research and engineering applications [3, 4, 22, 24], including nonlinear evolution equations in nonlinear physics [6, 8, 9, 25].

An important one-way nonlinear evolution equation is the Sawada-Kotera (SK) equation

$$u_t = u_{xxxxx} + 5u_x u_{xx} + 5uu_{xxx} + 5u^2 u_{xx},$$

where $u(x,t)$ is the field function of rescaled spatial and time variables x and t, cf. [35]. The SK equation explores the nonlinear behavior of waves in shallow water, ion-acoustic

*Corresponding author. *Email addresses:* gmwei@buaa.edu.cn (G.M. Wei), 13862416632@163.com (H.Q. Chen)

waves in plasma environments and fluid flow dynamics, describing the elevation of the water's free surface relative to a flat bottom [1, 16, 26].

The (2+1)-dimensional Sawada-Kotera equation

$$u_t = u_{xxxxx} + 5u_x u_{xx} + 5uu_{xxx} + 5u^2 u_{xx} + 5u_{xxy} - 5\partial_x^{-1} u_{yy} + 5uu_y + 5u_x \partial_x^{-1} u_y,$$

where ∂_x^{-1} is an integral operator with respect to x such that $\partial_x^{-1}\partial_x = \partial_x \partial_x^{-1} = 1$ was presented by Konopelchenko and Dubrovsky [18]. It is related to the inverse scattering transform method and can explain the long-wave phenomena [2, 17, 35]. It can also be written as

$$\begin{aligned} &u_t - u_{xxxxx} - 5u_x u_{xx} - 5uu_{xxx} - 5u^2 u_{xx} - 5u_{xxy} + 5v_y - 5uu_y - 5u_x v = 0, \\ &u_y - v_x = 0 \end{aligned}$$

with real differentiable functions $u(x, y, t)$ and $v(x, y, t)$ of three variables x, y and t [10]. The (2+1)-dimensional SK equation is a part of the Liouville field hierarchy, which plays a crucial role in quantum mechanics, plasma dynamics and nonlinear optics [1, 26].

The (2+1)-dimensional SK equation has been extensively studied. Using the Riccati equation, the Darboux and Bäclund transformations of the (2+1)-dimensional SK equation were derived, leading to the identification of rational, periodic and soliton solutions by Ma and Geng [23]. A novel approach employed in [27] allowed to explore a variety of traveling wave solutions of the (2+1)-dimensional SK equation. Through homoclinic test approach, lump, solitary, lump-stripe, and breather wave solutions were obtained [2]. Dual-wave solutions, multi-soliton solutions and new wave structures were acquired through different methods [15, 19, 21]. Spatial self-bending soliton phenomenon, nonlinear waves, and transitions mechanisms have been also studied — cf. [26, 36].

On the other hand, for uneven media and irregular boundaries, the models with variable coefficients provide a more accurate representations than the ones with constant coefficients [7, 11, 37]. In this paper, we focus on the following (2+1)-dimensional variable-coefficient Sawada-Kotera system:

$$\begin{aligned} &\alpha(t)u_t + \beta(t)\left(u_{xxxx} + 5uu_{xx} + \frac{5}{3}u^3\right)_x + \lambda(t)v_y + \gamma(t)\left(u_{xxy} + uv_x + u_x v\right) = 0, \\ &u_y - v_x = 0, \end{aligned} \tag{1.1}$$

where $\alpha(t), \beta(t), \lambda(t), \gamma(t)$ are real differentiable functions of t [10, 12, 35]. Note that the system (1.1), which can be also written as

$$\alpha(t)u_t + \beta(t)\left(u_{xxxx} + 5uu_{xx} + \frac{5}{3}u^3\right)_x + \lambda(t)\partial_x^{-1}u_{yy} + \gamma(t)\left(u_{xxy} + uu_y + u_x\partial_x^{-1}u_y\right) = 0,$$

plays an important role in shallow water waves, ion-acoustic waves in plasma physics and fluid dynamics [13]. Therefore, it has been vigorously studied. In particular, the breathers, lumps, line solitons and their interaction solutions were discussed in [35]. Two auto-Bäclund transformations and some solitons with all existing variable coefficients are given