

A Neural Network Modeling for MHD–Radiative Natural Convection Williamson Fluid Between Concentric Cylinders

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Abstract. This study investigates the natural convection flow of Williamson fluid between two concentric cylinders while affected by the radiation effect and magnetic field. The inner cylinder remains fixed while the outer cylinder rotates. Additionally, magnetic field is oriented radially, which influences the flow of the fluid. Applying a proper transformation, one transform the non-linear partial differential equations of the Williamson fluid model into ordinary differential equations. Artificial neural networks (ANN) facilitate the computation of solutions to these nonlinear ordinary differential equations. Trial functions employ a multilayer perceptron neural network with tunable parameters, including weights and biases. The governing equations are satisfied by determining the trial solution's changeable parameters by applying the Adam (adaptive moment estimation algorithm) optimization technique. Compared to the analytical solutions, the ANN's result demonstrates good accuracy. Moreover, graphs show how pertinent parameters affect the velocity and temperature profiles. The temperature and velocity profiles get smaller as the magnetic parameter value increases. Furthermore, the temperature and velocity profiles increase as the Hall parameter value rises.

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Key words: Artificial neural network, concentric cylinders, magnetic field, natural convection, radiation effect.

1. Introduction

The investigation of natural convection in the annular space between two vertically aligned concentric cylinders has gained significant attention due to its wide range of practical applications. The annular space between concentric cylinders can be utilized as a heat exchanger. Natural convection within this space helps to transfer heat between the cylinders making it applicable in cooling systems for electronics, industrial processes, growth

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of single silicon crystals, or other thermal management applications [24, 26, 43, 48]. Jha and Oni [19] investigated how the time-varying free convection flow between vertical concentric cylinders is influenced by the asymmetric heating or cooling of the surfaces of the cylinders. Ma *et al.* [33] numerically analyzed the free convective flow within a region bounded by two horizontal adiabatic coaxial cylinders, using three distinct nanofluids. Az-zawi *et al.* [7] studied numerically the free convection behavior of different nanofluids within both concentric and eccentric annular cavities subjected to internal heat flux and filled with porous media. Hameed *et al.* [13] examined the free convection heat transfer and fluid flow characteristics of a Casson-based nanofluid within a semi-parabolic enclosure that includes a wavy corrugated cylinder utilizing the Galerkin finite element method.

In the past few decades, the examination of non-Newtonian fluids in the context of heat and mass transfer has assumed a crucial role. The application of non-Newtonian fluids flowing between two concentric cylinders is prevalent in various industrial and scientific contexts such as polymer processing, the oil and gas industry, the food industry, drilling muds, and biological applications [12, 21, 36, 49]. Casson fluids, viscoelastic fluids, Jeffrey fluids, and Williamson fluids represent distinct categories of non-Newtonian fluids. Among these, the Williamson fluid is a special fluid for its unique viscoelastic characteristics within the realm of non-Newtonian fluids. The Williamson fluid was initially introduced by Williamson [51]. This particular fluid type was initially introduced to characterize pseudoplastic models that lack a genuine yield value and cannot be accurately modeled as either plastic or Newtonian fluids. Loganathan and Dhivya [31] examined a numerical investigation to analyze the boundary layer flow of a Williamson fluid passing a permeable vertical cylinder. Song *et al.* [44] investigated the unsteady, incompressible flow of Williamson nano liquid around a permeable stretching cylinder, considering variable thermal characteristics. The study explored how these factors interacted to influence the flow behavior. Zaman *et al.* [54] studied the flow of Williamson nanofluid under the influence of radiation effects around a slender cylinder. Recently, Kaladhar and Ravi [22] explored the irreversible phenomena in a Jeffrey fluid under mixed convection Navier-slip flow within a vertical channel, impacted by an inclined magnetic field. Ahad *et al.* [3] investigated the magnetohydrodynamic double diffusion of hybrid non-Newtonian ferro-fluid within a concentric corrugated cylinder with variable thermophysical properties using a machine learning approach.

The study of non-Newtonian fluids, particularly Williamson fluids, is of significant interest due to their unique rheological properties and wide range of applications. Williamson fluid belongs to the pseudoplastic fluid family. In [51], Williamson presented the concept of Williamson fluid during his initial work on pseudoplastic material flow. Williamson fluids exhibit shear-thinning behavior, meaning their viscosity decreases with increasing shear rate. He discussed in detail the pseudoplastic materials and then proposed a model with a compact set of equations to describe their flow, which was experimentally validated afterward. Now this model is known as the Williamson fluid model. The industrial and biological liquids that obey the Williamson fluid are polymers, melts, ketchup blood, paint, whipped cream, etc. Nadeem *et al.* [38] examined the two-dimensional flow of the Williamson fluid model over a stretching sheet solved with the help of the homotopy analysis method. Hashim *et al.* [14] studied the dual solutions incorporating the impacts of variable mag-