THE CONVERGENCE OF CONTOUR DYNAMICS METHODS * 1)

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Abstract

In this paper the properties of contour dynamics methods of two-dimensional incompressible inviscid vortex flows are investigated. The error estimates and the convergence of the methods for piecewise constant vorticity patches using Euler's method are obained.

1. Introduction

The numerical methods of vortex flows have attracted much attention. Since 1970 the vortex methods for numerical simulation of incompressible vortex flows have been greatly developed, for example, the random vortex methods [1], vortex in cell or cloud in cell methods [1], the particle methods [2] and contour dynamics methods [3]. For two-dimensional incompressible inviscid flows with initial piecewise constant vorticity patches it is sufficient to track the boundaries of the patches for simulating the evolution of the vortex flows. Hence, N. Zabusky et al. [3] proposed the contour dynamics code for simulating the vortex motion of flows with piecewise constant vorticity blobs, and numerically revealed a number of phenomena of vortex flows. Today many scientists are studying the problems in physics and fluid mechanics by means of the contour dynamics methods.

Up to now, due to the complexity of the evolution of the vortex, the problems of stability, convergence and the error estimates of the contour dynamics methods have not been discussed yet. This paper is one of the series of our works on analysis of contour dynamics methods. In section 2, we give a preliminary investigation of the flow motion. In section 3, the physical models of our consideration are proposed, and several conditions on the behaviour of the contour are imposed. Sections 4 and 5 are devoted to the truncation error and the convergence problems of the contour dynamics methods. Finally a discussion on the results obtained is given.

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* Received August 8, 1986.

1) The Project Supported by National Natural Science Foundation of China.

2. Preliminary considerations

In the conventional notations the inviscid incompressible vortex motions in two dimensions are described by the Helmholtz equations :

$$\begin{aligned}
&\frac{\partial \omega}{\partial t}+u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}=0,\\
&\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}=-\omega,\\
&\omega=-\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x},\\
&U=(u,v)^T=\left(\frac{\partial \psi}{\partial y},-\frac{\partial \psi}{\partial x}\right)^T.
\end{aligned} \tag{2.1}$$

$u, v(x,y,t)$ are the velocity components of the fluid particles, and $\omega(x,y,t)$ is called the vorticity density of the flow.

From (2.1) we have the integrals for the stream function ψ

$$\psi(x,y,t)=-\frac{1}{2\pi}\iint \ln r\cdot\omega(\xi,\eta,t)d\xi d\eta$$

and the velocity U

$$U(x,y,t)=\frac{1}{2\pi}\iint\begin{pmatrix}-\frac{\partial}{\partial y}\\ \frac{\partial}{\partial x}\end{pmatrix}\ln r\cdot\omega(\xi,\eta,t)d\xi d\eta$$

where $r^2=(x-\xi)^2+(y-\eta)^2$.

Let

$$z=(x,y)^T, z'=(\xi,\eta)^T,$$

$$K=K(z-z')=\frac{1}{2\pi}\left(-\frac{\partial}{\partial y},\frac{\partial}{\partial x}\right)^T\ln r=\frac{1}{2\pi}\left(-\frac{y-\eta}{r^2},\frac{x-\xi}{r^2}\right)^T.$$

We express the velocity as a convolution of K and ω,

$$U(z)=K*\omega=\iint_{R^2}K(z-z')\omega(z')dz'.$$

Euler's equation can be written as

$$\dot{z}=U(z).$$

Let $\omega^t(x,y)=\omega(x,y,t)$ be the vorticity function and $z(s,t)$ the contour equation at time t and $z_0(s)=z(s,0)$, where s is an arc parameter of the contour.

In what follows we use the notation $|\cdot|$ to denote the absolute value when it operates on a sclar and to denote the Euclidian norm when it operates on a vector.