Journal of Computational Mathematics, Vol.14, No.3, 1996, 256–262.

δ-WAVE FOR 1-D AND 2-D HYPERBOLIC SYSTEMS*1)

S.L. Yang
(*Institute of Applied Mathematics, Chinese Academy of Sciences, Beijing, China*)

Abstract

Here a new kind of nonlinear wave, which is called $\delta-$wave, is described by some high resolution difference solutions for Riemann problems of one-dimensional (1-D) and two-dimensional (2-D) nonlinear hyperbolic systems in conservation laws. Some phenomena are numerically shown for the solutions of Riemann problems for 2-D gas dynamics systems.

1. Introduction

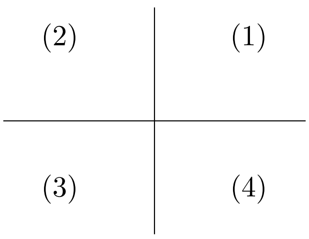
It is well known that hyperbolic systems of conservation laws have studied for a long time. By classical theoretical analyses, one classified the solutions to contain some natural sources such as constant, shock waves, rarefaction waves and contact discontinuities, which coincide with the solutions of practical problems, gas dynamics systems. In the recent years, we have studied for initial value problems of the following 2-D 2×2 hyperbolic systems[1,2,3,4],

$$\begin{cases} u_t + (u^2)_x + (uv)_y = 0 \ , \\ v_t + (uv)_x + (v^2)_y = 0 \end{cases} \tag{1.1}$$

with the 2-D Riemann data

$$(u, v)|_{t=0} = (u_0^i, v_0^i), \quad (i) = 1, 2, 3, 4 \tag{1.2}$$

where (i)-states are described to



* Received September 26, 1994.
1) The Project Supported by National Natural Science Foundation of China.

Here (1.1) is called 2-D inviscid Burger's equations.

For some distributions of Riemann initial data, a new kind of phenomenon was firs discovered by numerical simulations, that is, there are a narrow region near shock waves that solutions may produce infinity even though the initial data are bounded[1]. From the 2-D model, we can go back to some 1-D cases, then we consider the 1-D 2×2 nonlinear hyperbolic systems in conservation laws,

$$\begin{cases} u_t + f(u)_x = 0 \ , \\ v_t + (uv)_x = 0 \end{cases} \tag{1.3}$$

with initial data

$$(u, v)|_{t=0} = (u_0(x), v_0(x)) \ . \tag{1.4}$$

From the structure of (1.3), obviously, we can know that solution $v(x,t)$ may produce infinities if solution $u(x,t)$ has discontinuities.

In the coming sections, we will list some simple systems of (1.3) and give brief analyses and numerical solutions which contain $\delta-$waves for Riemann problems of 2-D 2×2 nonlinear hyperbolic systems in conservation laws. Finally we will show some singular phenomena for 2-D gas dynamic systems by numerical computations.

2. One Dimensional 2×2 Hyperbolic Systems

From (1.3), here we first choose f(u)=au and g(v)=b, then we have

$$\begin{cases} u_t + au_x = 0 \ . \\ v_t + bu_x = 0 \end{cases} \tag{2.1}$$

where a and b are both constants, and with initial data

$$(u, v)|_{t=0} = (u_0(x), v_0(x)). \tag{2.2}$$

From (2.1), the solutions of (2.1) and (2.2) can easily be obtained in the following forms,

$$\begin{cases} u(x,t) = u_0(x - at), \\ v(x,t) = v_0(x) - b\int_0^t (u_0(x - at))_x dt \ . \end{cases} \tag{2.3}$$

As the solution u(x,t) is obtained by a scalar equation which is the first equation of system (2.1), then obviously, the solution $v(x,t)$ will produce discontinuities if $u(x,t)$ containes discontinuities. However there is no more singular. Numerical description can be seen in [5].

Now we consider the following 2×2 nonlinear hyperbolic systems in conservation laws

$$\begin{cases} u_t + (u^2)_x = 0, \\ v_t + (uv)_x = 0 \end{cases} \tag{2.4}$$

and with initial data (2.2).