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A NEW GENERALIZED ASYNCHRONOUS PARALLEL MULTISPLITTING ITERATION METHOD*1)

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Abstract

For the large sparse systems of linear and nonlinear equations, a new class of generalized asynchronous parallel multisplitting iterative method is presented, and its convergence theory is established under suitable conditions. This method not only unifies the discussions of various existing asynchronous multisplitting iterations, but also affords new algorithmic and theoretical results for the parallel solution of large sparse system of linear equations. Besides its generality, this method is also much more suitable for implementing on the MIMD multiprocessor systems.

Key words: Systems of linear and nonlinear equations, Asynchronous multisplitting iteration, Relaxed method, Convergence theory.

1. Introduction

To solve large sparse systems of linear and nonlinear equations on the multiprocessor systems, many authors presented and studied various parallel iterative methods in the sense of multisplitting in recent years. For details one can refer to [1]-[9] and references therein. Among these methods the chaotic multisplitting iterative methods proposed by Bru, Elsner and Neumann[4] are meaningful on both theory and application since it aims at avoiding the synchronous wait among processors of a multiprocessor system and making use of the efficiency of the MIMD parallel computer. However, because more restrictions are imposed upon these chaotic multisplitting iterative methods (see [7,6]), the maximum efficiency in exploiting the resources of the multiprocessor systems has not yet been attained. To overcome this shortcoming, Evans, Wang and Bai (see [2,7]) further modified and developed Bru, Elsner and Neumann's work from the angles of both algorithmic model and theoretical analysis, and presented a series of asynchronous parallel multisplitting iterative methods. Recently, Su[6] also presented another generalization of Bru, Elsner and Neumann's chaotic multisplitting methods, which is called as generalized multisplitting asynchronous iteration. Since the designs of these asynchronous multisplitting methods take into account not only the good parallelism of the multiple splittings, but also the concrete characteristics of the multiprocessor systems, they can sufficiently exploit the parallel computational efficiency of the multiprocessor systems.

In this paper, by summarizing the advantages of the aforementioned asynchronous multisplitting iteration methods, we propose a new asynchronous parallel iterative method in the sense of multiple splittings, called as a new generalized asynchronous

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multisplitting iterative method (GAMI-method), for solving large sparse systems of linear and nonlinear equations. This new method has the properties of convenient implementation, and flexible and free communication, etc., and can also make full use of the efficiency of the multiprocessor systems. Meanwhile, the above stated existing asynchronous parallel multisplitting iterative methods are its special cases. Under similar conditions to [7] and [6], we establish convergence theory for our new method.

Since a system of equations can be equivalently transformed to several fixed point equations having a common fixed point by the multisplitting technique under certain conditions, without loss of generality, in the sequal we will mainly consider the iteration for getting a common fixed point of an operator class.

2. Description of the GAMI-Method

To mathematically describe our new generalized asynchronous multisplitting iterative method for parallely solving system of equations, we first introduce the following notations and concept.

Assume $\alpha(1 \leq \alpha \leq n)$ be a given positive integer. For all $i \in \{1, 2, \cdots, \alpha\}$, let $T_{p,i} : R^n \to R^n (p = 0, 1, 2, \cdots)$ be mappings having a common fixed point $x^* \in R^n$, and E_i be nonnegative, nonzero, diagonal matrices satisfying $\sum\limits_{i=1}^{\alpha} E_i = E$ nonsingular. Denote $N_0 := \{0, 1, 2, \cdots\}$ and $\mathcal{O}_T = \{T_{p,i} : R^n \to R^n \mid i \in \{1, 2, \cdots, \alpha\}; p \in N_0\}$. For any $p \in N_0$, we let $J(p)$ be a nonempty subset of the number set $\{1, 2, \cdots, \alpha\}$, and $s_j^{(i)}(p)$, $t_j^{(i)}(p) (j = 1, 2, \cdots, n; i = 1, 2, \cdots, \alpha)$ be nonnegative numbers satisfying:

(a) for $\forall i \in \{1, 2, \cdots, \alpha\}$, the set $\{p \in N_0 \mid i \in J(p)\}$ is infinite;

(b) for $\forall i \in \{1, 2, \cdots, \alpha\}$, $\forall j \in \{1, 2, \cdots, n\}$, $\forall p \in N_0$, there hold $s_j^{(i)}(p) \leq p$ and $t_j^{(i)}(p) \leq p$;

(c) for $\forall i \in \{1, 2, \cdots, \alpha\}$, $\forall j \in \{1, 2, \cdots, n\}$, there hold $\lim\limits_{p \to \infty} s_j^{(i)}(p) = \infty$ and $\lim\limits_{p \to \infty} t_j^{(i)}(p) = \infty$.

If we additionally define

$$\tau(p) = \min_{\substack{1 \leq j \leq n \\ 1 \leq i \leq \alpha}} \left\{ s_j^{(i)}(p), \quad t_j^{(i)}(p) \right\},$$

then there obviously have $\tau(p) \leq p$ and $\lim\limits_{p \to \infty} \tau(p) = \infty$.

With the above preparations, we can now describe the generalized asynchronous multisplitting iterative method (GAMI-method) for parallely solving systems of equations as follows.

GAMI-method. *Given an initial vector $x^0 \in R^n$, and suppose that we have got approximations $x^1, x^2, \cdots, x^p$ of a common fixed point $x^* \in R^n$ of the operator class $\mathcal{O}_T = \{T_{p,i} : R^n \to R^n \mid i \in \{1, 2, \cdots, \alpha\}; p \in N_0\}$. Then the next approximation x^{p+1} of x^* can be got by the following formula:*

$$x^{p+1} = \sum_{i \in J(p)} E_i T_{p,i} \left(x^{s^{(i)}(p)} \right) + \left(I - \sum_{i \in J(p)} E_i \right) x^p + \sum_{i \in J(p)} (I - E^{-1}) E_i \left(x^p - x^{t^{(i)}(p)} \right), \tag{2.1}$$

where