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A NEW TRUST-REGION ALGORITHM FOR FINITE MINIMAX PROBLEM*

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Abstract

In this paper, a new trust region algorithm for minimax optimization problems is proposed, which solves only one quadratic subproblem based on a new approximation model at each iteration. The approach is different with the traditional algorithms that usually require to solve two quadratic subproblems. Moreover, to avoid Maratos effect, the nonmonotone strategy is employed. The analysis shows that, under standard conditions, the algorithm has global and superlinear convergence. Preliminary numerical experiments are conducted to show the effiency of the new method.

Mathematics subject classification: 90C47, 49K35.
Key words: Trust-region methods, Minimax optimization, Nonmonotone strategy, Global convergence, Superlinear convergence.

1. Introduction

In this paper, we study the finite minimax problem of the form

$$(P):\min_{x\in\mathbf{R}^n}\max_{1\le i\le m} f_i(x), \tag{1.1}$$

where $f_i:\mathbf{R}^n\to\mathbf{R}$ are twice continuously differentiable. Many problems of interest in real world applications can be modeled as finite minimax problems (P). This class of problems occur, for instance, in curve fitting, $\mathbf{L}_1$ and $\mathbf{L}_\infty$ approximation problems, systems of nonlinear equations [1], problems finding feasible points of systems of inequalities, nonlinear programming problems, multiobjective problems, engineering design, optimal control and etc., which show that the finite minimax problem is a very important class of nonsmooth optimization problems. At present, many algorithms have been developed, which can be classified into two classes. One is that, the problem (P) can be viewed as an unconstrained nondifferentiable optimization problems, so we can use the methods for solving general nondifferentiable optimization problems, such as subgradient methods, bundle methods and cutting plane methods to solve it(see [2-7]). The other is that, in view of the particular structure of its nondifferentiability, it's also suitable to make use of smooth optimization methods which based on the well-known fact that the problem (P) is equivalent to a smooth optimization problem on the $n+1$ variables (x,z):

$$\begin{cases}\min\limits_{(x,z)\in\mathbf{R}^{n+1}} z,\\ s.t.\quad f_i(x)-z\le 0,\quad i=1,\cdots,m,\end{cases} \tag{1.2}$$

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where $x \in \mathbf{R}^n$ and $z \in \mathbf{R}$. Many line search algorithms were proposed by using this features (see [8-16]), under mild assumptions, these methods have good properties of both global convergence and locally superlinear convergence.

In history, most algorithms on minimax problems are line search rather than trust region based [17]. It's well known that trust-region methods are very efficient for smooth optimization problems, and they usually induce strong global convergence. Fletcher [18, 19] first applied trust-region methods to a class of composite nonsmooth optimization problems, and proposed a good trust-region algorithm, where the minimax problem can be regarded as its special case. Furthermore, Yuan [20] proved that it had a rate of superlinear convergence. However, it requires to compute the exact Hessian matrices $\nabla^2 f_i(x),\ i = 1, \cdots, m$ in the two subproblems at each iteration, which causes many gradient evaluations. Other trust-region methods that can be used to solve minimax problems were presented in [17], for example, Algorithm 11.3.1 thereof, but it is suitable to the general nonsmooth optimization problems.

Recently, the nonmonotone strategy has attracted attention from more and more researchers (see[21-24]), since Panier and Tits [21] indicated that, as an improvement of strategy of monotonic relaxation, it can prevent the Maratos effect when applied to the SQP algorithms for smooth optimization problems. Xue [8] proposed a new approximation model to the objective maximum function for minimax problems, which demonstrates some good properties. In addition, Wang and Zhang [25] proposed an algorithm based on trust region methods. Motivated by [8, 21], in this paper we develop a new trust region algorithm for finite minimax problems. On the one hand, unlike the line search methods such as [8–10, 15] in which the approximation Hessian matrices B_k have to be positive definite, the new algorithm does not require B_k to keep positive definite. On the other hand, the new algorithm is also different from [25]. First, it solves only one quadratic subproblem at each iteration. Second, it employs the nonmonotone strategy to overcome the Maratos effect. Third, it employs a new approximation model proposed in [8], and the corresponding subproblem is more stable. Under mild conditions, the global and superlinear convergence are obtained. Preliminary numerical experiments show that the proposed algorithm is robust and efficient.

The paper is organized as follows: In Section 2, we briefly recall trust-region methods; The algorithm is presented in Section 3. In Section 4, we analyze the global convergence and the rate of local convergence. In Section 5, we report some numerical results. Finally, we end the paper with conclusions.

We shall use the following notations and terminology. Unless otherwise stated, the vector norm used in this paper is Euclidean vector norm on $\mathbf{R}^n$, and the matrix norm is the induced operator norm on $\mathbf{R}^{n\times n}$. In addition, we denote

$$\phi(x) = \max_{1\le i\le m} f_i(x), \qquad \phi_k = \phi(x_k), \tag{1.3a}$$

$$I_A(x) = \Big\{i : f_i(x) = \phi(x)\Big\}, \qquad I_N(x) = \{i : f_i(x) < \phi(x)\}, \tag{1.3b}$$

$$f(x) = (f_1(x), \cdots, f_m(x))^T, \qquad \nabla f(x) = (\nabla f_1(x), \cdots, \nabla f_m(x)), \tag{1.3c}$$

$$F(x) = diag(f_1(x), \cdots, f_m(x)), \qquad e = (1, \cdots, 1)^T, \quad e \in \mathbf{R}^m. \tag{1.3d}$$

2. Trust Region Methods

Without loss of generality, in this section, we consider unconstrained optimization problem $\min_{x\in\mathbf{R}^n} f(x)$.