Journal of Computational Mathematics
Vol.42, No.6, 2024, 1427–1451.
http://www.global-sci.org/jcm
doi:10.4208/jcm.2309-m2023-0041

LOW-RANK MATRIX COMPLETION WITH POISSON OBSERVATIONS VIA NUCLEAR NORM AND TOTAL VARIATION CONSTRAINTS*

Duo Qiu
School of Mathematics and Physics, Wuhan Institute of Technology, Wuhan 430205, China
Email: qiuduoduo13@wit.edu.cn
Michael K. Ng
Department of Mathematics, Hong Kong Baptist University, Kowloon Tong, Hong Kong SAR, China
Email: michael-ng@hkbu.edu.hk
Xiongjun Zhang[1)]
School of Mathematics and Statistics and Hubei Key Laboratory of Mathematical Sciences, Central China Normal University, Wuhan 430079, China
Email: xjzhang@ccnu.edu.cn

Abstract

In this paper, we study the low-rank matrix completion problem with Poisson observations, where only partial entries are available and the observations are in the presence of Poisson noise. We propose a novel model composed of the Kullback-Leibler (KL) divergence by using the maximum likelihood estimation of Poisson noise, and total variation (TV) and nuclear norm constraints. Here the nuclear norm and TV constraints are utilized to explore the approximate low-rankness and piecewise smoothness of the underlying matrix, respectively. The advantage of these two constraints in the proposed model is that the low-rankness and piecewise smoothness of the underlying matrix can be exploited simultaneously, and they can be regularized for many real-world image data. An upper error bound of the estimator of the proposed model is established with high probability, which is not larger than that of only TV or nuclear norm constraint. To the best of our knowledge, this is the first work to utilize both low-rank and TV constraints with theoretical error bounds for matrix completion under Poisson observations. Extensive numerical examples on both synthetic data and real-world images are reported to corroborate the superiority of the proposed approach.

Mathematics subject classification: 15A83, 65K10, 90C30.
Key words: Low-rank matrix completion, Nuclear norm, Total variation, Poisson observations.

1. Introduction

The problem of low-rank matrix completion with Poisson observations is to estimate a low-rank matrix from given measurements at some subset of its locations, where the observations follow a Poisson distribution. Poisson observations appear in slew of practical applications in the areas of astronomical images, positron emission tomography, and magnetic resonance imaging [18, 33, 36]. Poisson noise is related to the count of photons recorded in the imaging devices, and can be modeled by a Poisson process, where the observations consist of counts of

* Received February 25, 2023 / Revised version received June 19, 2023 / Accepted September 8, 2023 /
1) Corresponding author

photons arrivals at a detector [50]. The total number of photons collected by imaging sensors follows a Poisson distribution. Furthermore, many real-world image data are low-rank or approximately low-rank. And during the data acquisition and processing procedures, only partial observations are available due to mechanical failure or human-induced factors. In this paper, we focus on the low-rank matrix completion problem with Poisson observations, where the entries one observes follow a Poisson distribution.

Low-rank matrix completion has received broad practical interest in past decades, and been applied in a variety of fields, such as image processing, computer vision, machine learning, and data mining [8, 12, 19, 22, 25, 27, 29, 38, 47]. For example, Candès *et al.* [4] presented a nuclear norm minimization method for matrix completion without noise and showed that one can recover a low-rank matrix exactly from a small number of its sampled entries with high probability. Moreover, Candès *et al.* [3] studied the matrix completion with additive noise and showed that the error of the estimator of the resulting model is proportional to the noise level. In particular, for the underlying matrix corrupted by sparse noise, it is possible to recover both the low-rank and the sparse components exactly with high probability under suitable assumptions by solving a very convenient convex program [2]. Besides, Taherkhani *et al.* [46] proposed a matrix completion approach based on nuclear norm minimization to predict the missing label of unsupervised nodes for graph-based semi-supervised learning, which trained a convolutional neural network based classifier using a large amount of unlabeled data and a small amount of labeled data. However, these methods did not consider the local prior information of the underlying matrix.

Being different from additive Gaussian noise or additive sparse noise, Poisson noise is nonadditive and signal-dependent. For the problem of compressed sensing with Poisson observations, Raginsky *et al.* [37] proposed a method consisting of a negative Poisson logarithmic likelihood term and a penalty term, and then established an upper bound of the estimator, where the penalty term was utilized to measure signal sparsity. Then Jiang *et al.* [24] provided minimax lower bounds on mean square errors for sparse Poisson inverse problems under nonnegative and flux-preserving constraints. Furthermore, Cao *et al.* [5] proposed a novel model composed of Kullback-Leibler divergence in the objective and the nuclear norm constraint for matrix completion with Poisson observations, and established an upper bound of the estimator of their proposed approach, which is minimax optimal up to a logarithmic factor. Soni *et al.* [45] proposed a novel model with unified framework for structural low-rank matrix completion with general noise observations, where the underlying matrix is factorized into the product of two matrices and one factor matrix is sparse. Then the error bounds of the estimator of the resulting model were established, where the minimax lower bounds of this kind of models were also derived in [42]. Recently, McRae *et al.* [32] proposed a low-rank matrix completion method by utilizing Frobenius norm for the data-fitting term, where both the nuclear norm constraint and nuclear norm regularized least squares were studied. However, the method in [32] just utilized the Frobenius norm for the data-fitting term, which is just suboptimal for Poisson observations [26]. Recently, Zhang *et al.* [54] proposed a transformed tensor nuclear norm constraint method for low-rank tensor completion with Poisson observations, while it only utilized the low-rankness of the underlying tensor and the local prior information was not considered.

In image restoration, some work discussed and studied grey image recovery with Poisson noise, where there are no missing entries for the observations, see, e.g. [6, 7, 15, 26, 30, 43, 49, 51, 53, 55, 56]. The main model in the literature composed of the KL divergence data-fitting term and the total variation (TV) regularization term. The TV regularization is proposed to