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ACCELERATED SYMMETRIC ADMM AND ITS APPLICATIONS IN LARGE-SCALE SIGNAL PROCESSING*

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Abstract

The alternating direction method of multipliers (ADMM) has been extensively investigated in the past decades for solving separable convex optimization problems, and surprisingly, it also performs efficiently for nonconvex programs. In this paper, we propose a symmetric ADMM based on acceleration techniques for a family of potentially nonsmooth and nonconvex programming problems with equality constraints, where the dual variables are updated twice with different stepsizes. Under proper assumptions instead of the so-called Kurdyka-Lojasiewicz inequality, convergence of the proposed algorithm as well as its pointwise iteration-complexity are analyzed in terms of the corresponding augmented Lagrangian function and the primal-dual residuals, respectively. Performance of our algorithm is verified by numerical examples corresponding to signal processing applications in sparse nonconvex/convex regularized minimization.

Mathematics subject classification: 47A30, 65Y20, 90C26, 90C90.
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1. Introduction

We consider a potentially nonsmooth and nonconvex separable optimization problem subject to linear equality constraints

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$$\begin{aligned} &\min\{f(\mathbf{x})+g(\mathbf{y})\} \\ &\text{s.t.}\quad A\mathbf{x}+B\mathbf{y}=b,\quad \mathbf{x}\in\mathcal{R}^m,\quad \mathbf{y}\in\mathcal{R}^n, \end{aligned} \tag{1.1}$$

where $f:\mathcal{R}^m\to(-\infty,+\infty]$ is a proper lower semicontinuous function, $g:\mathcal{R}^n\to(-\infty,+\infty)$ is a continuously differentiable function with its gradient ∇g being L_g-Lipschitz continuous, $A\in\mathcal{R}^{l\times m}, B\in\mathcal{R}^{l\times n}, b\in\mathcal{R}^l$ are given matrices and vector, respectively. Minimization problems of the form (1.1) appear in various applications in science and engineering. For example, the following l_1-regularized problem arises in signal processing and statistical learning [4, 5, 28]:

$$\min_{x\in\mathcal{R}^m}\frac{1}{2}\|A\mathbf{x}-c\|^2+\mu\|\mathbf{x}\|_1, \tag{1.2}$$

where $c\in\mathcal{R}^l$ is the observation vector, $A\in\mathcal{R}^{l\times m}$ is the data matrix and $\mu>0$ denotes the regularization parameter and is often set as $\mu=0.1\mu_{\max}$, where $\mu_{\max}=\|A^\mathsf{T}c\|_\infty$ (see e.g. [12, 28]). Due to the convexity of the problem (1.2), it can be handled by a number of standard methods, to list a few, including the alternating direction method of multipliers (ADMM, [10, 13, 14]), proximal point algorithm [4, 10], interior point method [28] and primal-dual hybrid gradient method [2, 45]. However, in many cases the l_1-regularization has been shown to be sub-optimal. For instance, it cannot recover a signal with the fewest measurements when applied in compressed sensing [7]. Therefore, an acceptable improvement is to adopt the $l_{1/2}$-regularization term, which results in the following problem:

$$\min_{x\in\mathcal{R}^m}\frac{1}{2}\|A\mathbf{x}-c\|^2+\mu\|\mathbf{x}\|_{\frac{1}{2}}^{\frac{1}{2}}, \tag{1.3}$$

where

$$\|\mathbf{x}\|_{\frac{1}{2}}=\left(\sum_{i=1}^n|\mathbf{x}_i|^{\frac{1}{2}}\right)^2$$

is a nonconvex function characterizing the sparsity, and it has been verified [42] practically to be better than l_1-norm. Clearly, by introducing an auxiliary variable, the above problem (1.3) can be converted to a special case of (1.1), i.e.

$$\begin{aligned} &\min\left\{\mu\|\mathbf{x}\|_{\frac{1}{2}}^{\frac{1}{2}}+\frac{1}{2}\|\mathbf{y}-c\|^2\right\} \\ &\text{s.t.}\quad A\mathbf{x}-\mathbf{y}=\mathbf{0}. \end{aligned} \tag{1.4}$$

The bold $\mathbf{0}$ denotes zero vector or matrix with proper dimensions. Another interesting example is the regularized empirical risk minimization arising from big data applications, such as many kinds of classification and regression models in machine learning [37, 41]. The $l_{1/2}$-regularized reformulation is of the form

$$\begin{aligned} &\min\left\{\mu\|\mathbf{x}\|_{\frac{1}{2}}^{\frac{1}{2}}+\frac{1}{N}\sum_{j=1}^N g_j(\mathbf{y})\right\} \\ &\text{s.t.}\quad \mathbf{x}-\mathbf{y}=\mathbf{0}, \end{aligned} \tag{1.5}$$

where N is a large number, $g_j(\mathbf{y})=\log(1+\exp(-b_i a_i^\mathsf{T}\mathbf{y}))$ denotes the logistic loss function on the feature-label pair (a_j,b_j) with $a_j\in\mathcal{R}^l$ and $b_j\in\{-1,1\}$.

In the literature, the most standard approach for solving the equality constrained problem (1.1) is the augmented Lagrangian method (ALM) which firstly solves a joint minimization