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CHAOTIC SYNCHRONIZATION OF QUATERNIONIC NEURAL NETWORKS WITH TIME-VARYING DELAYS AND ITS APPLICATION IN CRYPTOSYSTEM*

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Abstract

For complex-valued or quaternionic neural networks, scholars and researchers usually decompose them into real-valued systems. The decomposed real-valued systems are equivalent to original systems. Then, the dynamical behaviors of real-valued systems obtained are investigated, including stability, synchronization, and chaos etc. In this paper, a class of quaternionic neural networks with time-varying delays is investigated. First, by designing a suitable PI controller, synchronization of the considered chaotic system is realized. By using a non-decomposition method and structuring a novel Lyapunov functional, sufficient conditions are derived to guarantee synchronization between the drive-response systems. It is worth mentioning that, unlike other methods, our approach does not require breaking down the quaternionic neural networks into four separate real-valued systems. Furthermore, we demonstrate the practical application of these chaotic quaternionic neural networks with time-varying delays in image encryption and decryption. Based on one sequence of chaotic signal from state trajectory of single quaternion-valued neuron and a new encryption algorithm, the application of chaotic system proposed, that is, image encryption, is researched. The process of image decryption is simply the reverse of the encryption process. Finally, numerical simulation examples are provided to validate the effectiveness of the designed PI controller and performance of image encryption and decryption.

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1. Introduction

Quaternions, as a type of hypercomplex, have been successfully applied in many fields, such as computer vision, crystallographic texture and quantum mechanics [11,17,47], etc. Quaternion can efficiently reduce the computational complexity of high-dimensional problems [6]. In view of the advantages of quaternion, some scholars and researchers propose and model quaternionic neural networks (QNNs). QNNs have significant applications in color image compression, color night vision, robots and other fields [31, 33, 40, 48, 54]. Compared with complex-valued neural networks (CVNNs), QNNs with higher storage capacity can directly deal with 3D and 4D problems which can also be solved with a great number of neurons in CVNNs [8,10,21,22,34,48]. So far, researches on the dynamics of QNNs have made a lot of achievements [9,12,15,43], just to cite a few. Chen *et al.* [9] investigated the problem of stability for continuous-time and discrete-time QNNs. In [15, 24], stability for a delayed fractional-order QNNs was researched. The state estimation problem of QNN with time-varying delays (TVDs) was studied by extending Jensen's inequality to the quaternion domain [43]. In [12], authors studied the stability problem of a class of stochastic QNNs with TVDs using stochastic analysis techniques and Lyapunov functions (LF). In [51], the Mittag-Leffler stability and synchronization of Caputo fractional-order fuzzy QNNs with proportional delay and derivative order intervals were studied. In [20], by adopting an extended modification Lyapunov-Razumikhin method, global finite-time stability problem for QNNs was researched. In [23], exponential estimation and passivity of memristive neural networks (NNs) with quaternion parameters were studied. A Takagi-Sugeno type rule was introduced into the quaternion memristive NNs, which makes the system much easier. In [39], by using non-separation method, the global asymptotic stability problem for Takagi-Sugeno fuzzy quaternion-valued bidirectional associative memory NNs with discrete, distributed and leakage delays was investigated.

Proverbially, the signal transmission between neurons inevitably exists time delay [36, 42], which affect the dynamical behaviors of NNs, including chaos, oscillation, instability, etc. [7, 30, 35, 37, 38, 50]. For chaotic NNs, it has the characteristics of noise-like behavior and non-periodicity, and follows deterministic rules. Due to chaotic NNs are sensitive to initial conditions [49], small changes in initial conditions will have a significant impact on signal behavior. These characteristics make chaotic NNs good candidates for cryptosystem [19, 45]. At present, many results about chaotic NNs have obtained [1, 14, 32, 52, 53], just to cite a few. The asymptotic stability of transiently chaotic NNs is considered in [52]. Through observer based sliding mode control, Zhao *et al.* [53] studied the synchronization problem of delayed chaotic NNs with unknown disturbances. A secure communication scheme based on event triggered strategy and master-slave NNs quantization synchronization was proposed in [14]. The memristive chaotic NNs and the cascaded chaotic NNs were presented in [32] and [1], respectively.

With the increase in communication frequency on open networks, secure communication is becoming increasingly important. A good encryption process should be sensitive to cipher key, and the key space should be large enough to make brute-force attacks infeasible [2]. Since chaotic NNs have both the characteristic of NNs and chaos, chaotic NNs are applied to secure communication [3, 25]. Thus, the researches on dynamical properties of delayed chaotic NNs