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An Effective ℓ_p-Nonconvex Regularization Method for Image Smoothing

Biao Fang[1], Xiao-Guang Lv[2,*], Guo-Liang Zhu[1,3], Le Jiang[2] and Jia-Qi Mei[1]

[1] *School of Electronic Engineering, Jiangsu Ocean University, Lianyungang, Jiangsu 222005, China*
[2] *School of Science, Jiangsu Ocean University, Lianyungang, Jiangsu 222005, China*
[3] *Ocean College, Jiangsu University of Science and Technology, Zhenjiang, Jiangsu 212003, China*

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Abstract. Image smoothing techniques are widely used in computer vision and graphics applications. In this paper, we present an ℓ_p-nonconvex regularization method for image smoothing. To induce sparsity prior of the smoothed images more strongly than the ℓ_1 norm regularization, we take the nonconvex arctangent penalty function of the image gradient as the regularization term. To make the proposed model more flexible and effective for different image smoothing applications, we use the ℓ_p norm function as the fidelity term, instead of the ℓ_2 norm function. The powerful majorization-minimization (MM) algorithm is employed for the proposed nonconvex optimization model. The convergence of the resulting MM algorithm is discussed. Comprehensive experiments and comparisons show that the proposed method is effective in various image processing tasks such as texture smoothing, detail enhancement, artifact removal, image denoising, high dynamic range (HDR) tone mapping, edge detection, and image composition.

AMS subject classifications: 94A08, 68U10, 90C26

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1 Introduction

The purpose of image smoothing is to eliminate unwanted details while maintaining the important edges. As a significant image processing technique, image smoothing is

*Corresponding author.
Emails: biaofang2021@163.com (B. Fang), xiaoguanglv@jou.edu.cn (X. Lv), guoliangzhu2021@163.com (G. Zhu), jiangle@jou.edu.cn (L. Jiang), mjq1333907@163.com (J. Mei)

widely used in computer vision and graphics applications such as texture smoothing, detail enhancement, artifact removal, image denoising, high dynamic range (HDR) tone mapping, edge detection, and image composition [1–7]. In recent years, image smoothing methods have received considerable attention from the image processing research community.

Generally, edge-preserving methods are used to smooth out the small details while preserving or sharpening the strong edges. One of the most classical edge-preserving methods is the bilateral filter (BF) [8–10]. The BF is widely used in various smoothing tasks due to its simplicity and effectiveness in removing noise-like structures. However, as pointed out by He et al. in [11], the BF tends to produce undesirable gradient reversal artifacts at the edges. A guided filter was proposed by He et al. in [11] to alleviate this problem and obtain high-quality images. To reduce the running cost of BF, Zhang et al. in [12] designed a fast weighted median filter that runs more than 100 times faster than BF. In [13], to improve the ability of BF-based filters to extract the multi-scale details, Farbman et al. introduced the weighted least squares (WLS) optimization method. Although the WLS method alleviates many problems, it is insufficient in capturing fine-scale textures. To remedy this deficiency, Subr et al. in [14] explored the local extreme model to extract fine-scale features. This model captures textures based on the assumption that fine-scale details are rapidly oscillating information between minima and maxima. In [15], Gastal et al. considered a domain transformation method for addressing the gradient reversal problem by using one-dimensional space to accelerate two-dimensional edge-aware filtering. In [16], Xu et al. used the ℓ_0 gradient minimization (ℓ_0GM) method to enforce the gradient sparsity of the smoothed images. In [17], Paris et al. designed a local Laplacian filter for edge-aware image smoothing. The critical step is to characterize the edges with a simple threshold on the pixel values so that large-scale edges can be distinguished from small-scale details. In [18], Ham et al. developed a static and dynamic (SD) filter to handle structural inconsistencies between the guidance and input images. To improve the problem of over-sharpening caused by excessive regularization parameter in the ℓ_0-norm method, Liu et al. in [19] adopted the gradient sparsity and surface area (GSSA) minimization scheme. In [20], Liu et al. proposed an effective iterative least squares (ILS) for edge-preserving image smoothing. The implementation of the ILS algorithm is close to real-time due to the use of the Fourier transform during the iterative process. In [21], Zhu et al. built a benchmark for edge-preserving image smoothing from the point of both qualitative and quantitative performance evaluation. The reasonable edge-preserving filter shouldn't blur or over-sharpen the edges that define object boundaries or other important features, while smoothing the areas between these edges. Unfortunately, such a filter does not exist, as it is generally not possible to accurately determine which edges should be retained.

Structure-preserving smoothing methods are usually applied to preserve the salient structures of the restored images. In [22], Xu et al. developed the relative total variation (RTV) measures to accomplish structure extraction from texture images. In [23], Karacan et al. employed a novel approach based on patch-analysis. Specifically, they estimated