Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **17**, No. 6, pp. 1654-1681

DOI: 10.4208/aamm.OA-2024-0017
December 2025

Strong Converge Order of the General One-Step Method for Neutral Stochastic Delay Differential Equations under a Global Monotone Condition

Chao Yue[1,*], Haoyong Zhuang[1] and Longbin Zhao[2]

[1] *School of Economics, Zhengzhou University of Aeronautics, Zhengzhou, Henan 450015, China*
[2] *School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo, Henan 454000, China*

Received 24 January 2024; Accepted (in revised version) 26 July 2024

Abstract. We study the strong convergence of the general one-step method for neutral stochastic delay differential equations with a variable delay. First, we give the notions of C-stability and B-consistency, and then establish a fundamental theorem of strong convergence for the general one-step method solving the nonlinear neutral stochastic delay differential equations, where the corresponding diffusion coefficient with respect to the non-delay variables is highly nonlinear. Then, we construct the split-step backward Euler method which is a special implicit one-step method, and prove that it is C-stable, B-consistent, and strongly convergent of order 1/2. Finally, we give some numerical experiments to support the obtained results.

AMS subject classifications: 65C20, 65L20, 60H35

Key words: Strong convergence, general one-step method, C-stability, B-consistency, neutral stochastic delay differential equations.

1 Introduction

In recent years, neutral stochastic delay differential equations (NSDDEs) have been widely applied to neural network, physics, finance, mathematical biology and so on (see [1–4]). Since most NSDDEs cannot be solved explicitly, numerical methods have become more essential. One of the interesting problems in numerical methods is the investigation of the strong convergence. It was usually researched under a general assumption that the drift and diffusion coefficients fulfill global Lipschitz and linear growth conditions. Unfortunately, the coefficients of many important NSDDEs (see [2,5–7]) violate the

*Corresponding author.
Email: chao_yue@zua.edu.cn (C. Yue)

 ©2025 Global Science Press

assumption. So these classical strong convergence results (see [8, 9]) cannot be utilized. Thus, many researchers turn to studying strong convergence of numerical methods for the NSDDEs with the non-global Lipschitz continuous coefficients.

Some explicit methods have been constructed to solve the NSDDEs satisfying the non-global Lipschitz continuous conditons. For example, under the local Lipschitz and linear growth conditions, Wu and Mao [10] obtained the strong mean-square convergence of the Euler-Maruyama (EM) method for neutral stochastic functional differential equations (NSFDEs). Furthermore, Zhou and Wu [11] applied the results to the NSDDEs with Markovian switching. Under the highly nonlinear conditions, Milošević [6] and Zhou and Fang [12] studied the convergence in probability of the EM method for NSDDEs and NSFDEs, respectively. Ji and Yuan [13] investigated the strong convergence of the tamed EM method for solving NSDDEs with the coefficients satisfying the super-linear condition. Under local Lipschitz and Khasminskii-type conditions, Lan and Wang [7] constructed the modified truncated EM method and obtained its strong convergence order for NSDDEs with a constant delay.

Some implicit methods have been devised for such problems as well. Under the highly nonlinear conditions, Milošević studied the convergence of the forward-backward methods in [14] and the backward Euler method in [15] in probability for NSDDEs, respectively. Zhou and Jin [16] proved the strong convergence of the backward Euler method for stochastic functional differential equations with superlinear growth coefficients, but did not get its strong convergence order. Under the assumption that the coefficients of NSDDEs may be highly nonlinear with respect to the delay variables, the split-step method is strongly convergent of order $1/2$ in [17] and the theta method is strongly convergent of order $1/2$ in [18]. In [19] a tamed theta scheme was constructed for NSDDEs with one-sided Lipschitz drift. However, under the non-global conditions, there are little results on the strong convergence order of numerical methods solving NSDDEs with a time-varying delay. Inspired by these works, in this paper, we further investigate the strong convergence order of numerical methods for NSDDEs satisfying the non-global conditions.

C-stability was first introduced in [20, 21]. Then Beyn, Isaak and Kruse [22, 23] applied it to stochastic differential equations and proposed B-consistency. Furthermore, Yue [24] and Li, Huang and Chen [25] extended them to stochastic differential equations with jumps, and Yue and Zhao [26] and Li and Huang [27] extended them to stochastic delay differential equations. In this paper, we further give the notions of C-stability and B-consistency for NSDDEs, and based on these conditions establish an important strong convergence theorem of the general one-step method (3.1) for the nonlinear NSDDEs with the non-global Lipschitz continuous coefficients, that is, the drift and diffusion coefficients fulfill a local Lipschitz condition and a coupled condition. These conditions allow the diffusion coefficient to be highly nonlinear with respect to the non-delay variables. Furthermore, we construct the the split-step backward Euler method (4.1) which is a special implicit scheme of the general one-step method (3.1), and prove that it is C-stable, B-consistent, and strongly convergent of order $1/2$. Compared to the existing