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A Fast Matrix Splitting Iteration Method for Fractional Regime-Switching Option Pricing Model

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Abstract. The discretization of the fractional regime-switching option pricing model leads to the linear system with the block diagonal and Kronecker product structure. A fast matrix splitting iteration method is presented to solve the discrete system. Theoretical analyses prove the convergence of the proposed iteration method. Numerical experiments show that the new method is efficient and outperforms the existing methods in terms of the number of iteration steps and the elapsed CPU time.

AMS subject classifications: 65M06, 65M22, 65M70, 91G60

Key words: Fractional options pricing, splitting iteration, convergence, spectral analysis.

1 Introduction

Assuming that the underlying asset follows a geometric Brownian motion, Black-Scholes model was firstly proposed in 1973 for pricing the options and quickly spread in the financial market. In order to better simulate the skewed and heavy-tailed return distributions of the underlying asset in the practical markets, stochastic volatility models [21], jump-diffusion models [14, 16] and fractional models including FMLS [7], CGMY [6] and Kobol [13] models were further proposed and deeply studied. In addition, regime-switching models [5,12] are established for pricing the options by incorporating the non-stationary behavior.

Since the exact solution of fractional option pricing model is usually difficult to be obtained or even does not exist, numerical approaches are proposed for computing the approximate solution of fractional partial difference equations [20, 23]. An approximate inverse preconditioned policy iteration method was applied for solving the fractional

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HJB equation governing American options models [9]. A penalty method with approximate inverse preconditioner was studied for the fractional regime-switching diffusion model [15]. An implicit-explicit preconditioned direct method was designed to price options under the fractional regime-switching processes by approximating Toeplitz matrix with circulant matrix [8].

The discrete fractional partial difference equations usually results in the linear systems with special structure, thus matrix splitting iteration methods attracted much more attention to quickly solve the linear equations. The diagonal and Toeplitz splitting iteration method was established for solving the discretized spatial fractional diffusion equations [4]. The banded M-splitting iteration method was presented by preserving the property of M-matrix [3]. The positive-definite operator splitting iteration method was constructed to solve the variable-coefficient space-fractional diffusion equations [22]. The idea of matrix splitting introduced good preconditioners for the Krylov subspace solver, see [17,18] for the details.

After the discretization of the fractional regime-switching option pricing model, the coefficient matrix has a structure of the sum of block diagonal matrix and Kronecker product matrix. Thereby, a new matrix splitting iteration method based on this special structure is established and studied. The convergence of it under sufficient conditions is proved. The implementations and acceleration of the proposed method are discussed in detail. Several experiments are performed to compare the numerical results using different splitting methods, and our numerical results show the high efficiency of this new approach compared to several previously proposed techniques.

The rest of this paper is organized as follows. In Section 2, the discretization of the tempered fractional partial differential equations is presented. In Section 3, we construct the block diagonal and Kronecker product splitting iteration method and analyze its convergence. In Section 4, numerical experiments are presented to verify the effectiveness of the proposed method. Finally, in Section 5, we end this paper with conclusions.

The standard notation is followed in this work. For example, $\mathbb{R}^n$ will be used to denote the set of n-dimensional column vectors. Matrix A with m rows and n columns belongs to $\mathbb{R}^{m\times n}$. A^T will be used to denote the transpose of matrix A, with diag(A) denoting the diagonal of matrix A. I_n will be used as the $n\times n$ identity matrix. $\langle x,y\rangle = x^Ty$ denotes the standard inner product and $\|x\| = \sqrt{\langle x,x\rangle}$ as the Euclidean (L_2) norm. $\|A\|_2$ is the 2-norm of the matrix A.

2 Discretization of the fractional regime-switching option pricing model

We consider the following fractional difference equations governing the fractional regime-