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A Hybrid Reconstruction Method Based upon the Characteristic Fields Decomposition of Compressible Euler Equations

Ke Zhang[1,2] and Yiqing Shen[1,2,*]

[1] *State Key Laboratory of High Temperature Gas Dynamics, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China*
[2] *School of Engineering Science, University of Chinese Academy of Sciences, Beijing 100049, China*

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Abstract. The local characteristic fields of compressible Euler equations can be decomposed into linearly degenerate and genuinely nonlinear, respectively. The former is associated with a contact discontinuity (the slip lines for multi-dimensional cases) and the latter is associated with shocks and rarefaction waves (Toro, Riemann Solvers and Numerical Methods for Fluid Dynamics, Springer, 2009). The weighted essentially non-oscillatory (WENO) scheme can capture shock waves well but its resolution for short waves still needs to be improved, while one kind of new scheme constructed by the boundary variation diminishing (BVD) principle with the THINC (Tangent of Hyperbola for INterface Capturing) reconstruction can resolve the contact discontinuities with higher resolution but it may generate spurious numerical phenomena in some special cases. Combining the physical characteristics and the advantages of WENO and BVD schemes, this paper proposes a hybrid reconstruction method, in which the classical WENO scheme is used to discretize the genuinely nonlinear fields and the higher resolution BVD scheme is used to discretize the linearly degenerate field(s). Numerical experiments show that the hybrid method can preserve high accuracy near both contact discontinuities and short waves as the used higher resolution scheme while effectively avoiding spurious numerical phenomena (such as oscillations and overshoots). Compared with the BVD scheme, the hybrid method also improves computational efficiency, since it uses a more efficient WENO scheme in genuinely nonlinear fields.

AMS subject classifications: 35L65, 65M08, 76M12

Key words: Characteristic fields decomposition, hybrid reconstruction, WENO scheme, BVD scheme, high resolution.

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*Corresponding author.
Email: yqshen@imech.ac.cn (Y. Shen)

1 Introduction

Complex flow features, such as the shock wave, contact wave, rarefaction wave, multi-scale structures, and their interactions, are challenging for accurate and efficient numerical methods of compressible flows. In general, shock-capturing methods commonly introduce sufficient dissipation to prevent numerical oscillations and spurious overshoots near discontinuities. But that in turn, inevitably affects their spectral properties for smooth solutions and decreases the resolution of multi-scale structures.

In the recent three decades, the weighted essentially non-oscillatory (WENO) scheme as one kind of shock-capturing schemes have been widely studied and applied in computational fluid dynamics, since they can keep the ENO property near shock waves and have uniformly high order accuracy in smooth regions. The first WENO scheme was proposed by Liu et al. [1] and then was improved by Jiang and Shu [2] by designing a classical formulation to calculate the smoothness indicators of sub-stencils used in the corresponding ENO schemes. Then many works are successively presented to analyze and improve the performances of the WENO schemes, such as the higher-order WENO schemes [3, 4], the WENO schemes with the improved accuracy at critical points [5–9], the multi-step WENO schemes with the improved accuracy at transitional points [10,11], and the TENO (targeted essentially non-oscillatory) schemes with lower dissipation and improved robustness [12–14].

In the complex compressible flow fields, contact discontinuity is another flow structure different to the shock wave, and many upwind methods which have high resolution for shock waves may not perform so good for contact discontinuities, such as the apparent smearings or even spurious numerical oscillations, and hence downwind methods were proposed [15–19]. Recently, Sun et al. [20] proposed BVD (boundary variation diminishing) principle for constructing high-fidelity discontinuity-capturing schemes with lower numerical dissipation. The BVD principle requires that the jumps of the reconstructed values at cell boundaries should be minimized so as to reduce the dissipation in Riemann solvers. Then, several BVD variant algorithms were devised. Among which, the $\mathrm{P_nT_m}$-BVD (polynomial of n-degree and THINC function of m-level reconstruction based on BVD algorithm) schemes [21–24] show an improved resolution for shock waves, contact discontinuities, and also the small-scale structures. One of the shortcomings of the $\mathrm{P_nT_m}$-BVD schemes is that they may generate spurious overshoots or undershoots in some special cases (see the Figs. 9 and 13 in [21] and the Figs. 5 and 6 in [22] as examples).

For discretizing a hyperbolic system, the use of characteristic variable can help to improve the shock-capturing capability and robustness. Based on the wave-propagation theory, the local characteristic fields of compressible Euler equations can be decomposed into genuinely nonlinear and linearly degenerate [25]. For the Riemann problem [26], the linearly degenerate field is associated with a contact discontinuity (entropy wave) and slip lines (shear wave), the genuinely nonlinear fields are associated with shocks (acoustic wave) and rarefaction waves. Considering the features of abovementioned two kinds of schemes (WENO and BVD), in this paper, we apply a hybrid method to validate