Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **17**, No. 6, pp. 1813-1840

DOI: 10.4208/aamm.OA-2023-0208
December 2025

Accurate Static and Dynamic Analysis Using Unstructured Meshes for Complicated Geometry Problems

Guangze Tang, Hong Yang, She Li, Xin Hu and Xiangyang Cui*

State Key Laboratory of Advanced Design and Manufacturing Technology for Vehicle, Hunan University, Changsha, Hunan 410082, China

Received 1 August 2023; Accepted (in revised version) 12 November 2023

Abstract. The low accuracy using unstructured meshes to solve complicated geometry problems is a significant challenge in practical engineering. It is the key to solve this issue through enhancing the accuracy of tetrahedral element. This work develops a simple strategy to enhance the accuracy of tetrahedral element with the Shepard interpolation function. In which the strain field is reconstructed by introducing the weighted strains of adjacent tetrahedral elements to central tetrahedral element. The stiffness of the discrete system is significantly softened with this simple operation, which leads to a great accuracy improvement. Furthermore, this simple modification makes linear tetrahedral element owns higher accuracy even compared with hexahedral element. The high precision tetrahedral element makes finite element analysis more acceptable for engineers. It provides a novel solution to finite element analysis using unstructured meshes for practical engineering problems with complicated geometry. In order to validate the accuracy and feasibility of present modified tetrahedral element (M-T4) in complicated geometry problems, several engineering examples are performed, including linear static analysis, modal analysis, frequency response analysis, transient response analysis, and response spectrum analysis. The remarkable performance of the M-T4 suggests its wide applicability to practical engineering problems.

AMS subject classifications: 65N30, 74H15

Key words: Finite element method, modified linear tetrahedral element, Shepard interpolation method, linear dynamic analysis, complicated geometry.

1 Introduction

In the wake of the rapid development of computer technology, numerical simulation has assumed an increasingly important role in the structural design of mechanical parts.

*Corresponding author.
Email: cuixy@hnu.edu.cn (X. Cui)

There are several popular methods available for this purpose, including the Finite Element Method (FEM) [1, 2], the Finite Volume Method (FVM) [3], the Boundary Element Method (BEM) [4, 5], and the Meshfree Method [6] etc.. Nonetheless, the FEM stands out as the most widely utilized method for solid and structural mechanical problems and has already been integrated into many advanced CAE software.

Compared with hexahedral elements, linear tetrahedral elements have some irreplaceable advantages, such as ease of division for complicated structures and strong anti-distortion ability. Nonetheless, due to the limited capacity of the linear displacement-based formulation of linear tetrahedral elements to accurately capture complex deformations and stress states, they exhibit "overly-stiff" behavior, leading to significant accuracy issues. As a result, research aimed at improving the accuracy of linear tetrahedral elements holds great practical value.

To address the limitations of the linear tetrahedral elements, researchers have dedicated significant effort towards developing effective alternatives, among which the smoothed finite element methods (S-FEMs) and the meshfree methods are the most famous. The smoothed finite element methods are developed based on the gradient smoothing technique which is proposed by Liu [7]. It has been proven that the S-FEMs can effectively soften the "overly-stiff" behavior of FEM models. As a result, they often yield more accurate results than the standard FEM. The node-based smoothed finite element method (NS-FEM) utilizing node-based strain smoothing operation have been successfully applied to analyze solid mechanics problems [8], acoustic problems [9] and others. However, because of its "overly-soft" behavior, the NS-FEM can result in temporal instability while solving dynamic problems, leading to inaccurate results [10]. To address this issue, Feng et al. [11] proposed a stable node-based smoothed finite element method (SNS-FEM) that intensifies the relatively soft NS-FEM and effectively eliminates the spurious non-zero energy modes. The effectiveness of the SNS-FEM for dynamic analysis has been proved by Hu et al. [12] and Cui et al. [13]. Based on the SNS-FEM, a modified stable node-based smoothed finite element method (M-SNS-FEM) is proposed by Feng et al. [14] to enhance the applicability and accuracy of linear low-quality unstructured mesh. Xuan et al. [15] extended NS-FEM to compute upper bound and lower bound, observing the advantages of it over the conventional method. Liu et al. introduced the edge-based smoothed finite element method (ES-FEM), which applies strain smoothing technique to the domain related to the element edge [16]. Numerical results [17-19] have demonstrated that the ES-FEM model possesses a stiffness approximation close to exact, striking a balance between the "overly-stiff" FEM and the "overly-soft" NS-FEM. Cao et al. [20] proposed a parallel computing scheme based on the ES-FEM, improving the calculation efficiency. The face-based smoothed finite element method (FS-FEM), applying strain smoothing technique to the domains associated with the faces of the tetrahedral elements, was presented by Nguyen-Thoi et al. [21]. He et al. [22] and Zhao et al. [23] demonstrated that the FS-FEM can provide more accurate results than the FEM for 3D dynamic solid mechanics, but also exhibits a slightly "overly-stiff" behavior. Yao et al. [24] developed a modified smoothed finite element method (M-