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Corrected Linear-Galerkin Schemes to Preserve Second-Order Accuracy for Cell-Centered Unstructured Finite Volume Methods

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Abstract. Unstructured finite volume methods are typically categorized based on control volumes into the node- and cell-centered types. Because of certain inherent geometric properties, the second-order Linear-Galerkin scheme, favored for its simplicity and ability to preserve second-order solution accuracy, is predominantly applied to node-centered control volumes. However, when directly applied to cell-centered control volumes, the designed solution accuracy can be lost, particularly on irregular grids. In this paper, the least-square based Linear-Galerkin discretization is extended to arbitrary cell-centered elements to ensure the second-order accuracy can always be achieved. Altogether four formulations of corrected schemes, with one being fully equivalent to a conventional second-order finite volume scheme, are proposed and examined by problems governed by the linear convective, Euler and Navier-Stokes equations. The results demonstrate that the second-order accuracy lost by the original Linear-Galerkin discretization can be recovered by corrected schemes on perturbed grids. In addition, shock waves and discontinuities can also be well captured by corrected schemes with the help of gradient limiter function.

AMS subject classifications: 35L55, 65M08, 76M12, 76M10

Key words: Unstructured finite volume methods, second-order accuracy, Linear-Galerkin scheme, corrected schemes, cell-centered control volume.

1 Introduction

Because of multiple advantages, including convenient implementation and excellent robustness on arbitrary girds, the unstructured finite volume (FV), especially the second-order methods are widely used and become a preferred choice [1–18]. FV methods are

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typically categorized based on control volumes into the node- and cell-centered types, with numerous studies focusing on their comparative analysis and the development of corresponding numerical algorithms [19–31].

Specifically, for node-centered elements, popular schemes include the Linear-Galerkin (LG) [32–35] and flux correction (FC) schemes [28, 33, 36–38]. Between them, the LG is a second-order scheme and will be introduced later, whereas the FC is third-order accurate. Different from quadrature-based high-order FV methods [39–46], for the FC scheme, the differential conservation law is directly discretized, and a single quadrature point at per edge is enough to preserve the third-order accuracy. It was initially proposed by Katz & Sanakaran [33] and further developed by Nishikawa, including accuracy preserving boundary quadrature [47], compatible source term discretization [48, 49], as well as extensions to hyperbolic system [50–53] and unsteady problems [54].

The main focus of this paper is on schemes preserving second-order accuracy, and thus, high-order discretization will not be extensively covered. As mentioned before, the LG scheme is widely used on the node-centered control volume since the flux quadrature point is exactly halfway between two adjacent local origins, and from numerical results given in [32], the second-order solution accuracy can be preserved on arbitrary grids. However, this geometric property is not inherently present in cell-centered elements. Therefore, if the LG discretization is directly applied, solution accuracy can be reduced to the first-order. The similar problem appears in the U-MUSCL scheme [55, 56] with a nonzero parameter χ, but ingenious correction strategies named LP-UMUSCL were proposed by Nishikawa [57]. With this method, the solution is finally upgraded from the first- to second-order accurate.

By analogy with the LG and greatly inspired by the LP-UMUSCL scheme, a series of corrected Linear-Galerkin (CLG) schemes that preserve second-order accuracy on arbitrary cell-centered elements are developed in this paper. Interestinigly, one of these schemes is exactly equivalent to a conventional second-order cell-centered FV scheme. Thus, in this regard, a more generalized framework for that is provided.

The paper is organized as follows. In Section 2, a conventional second-order FV discretization for the integral conservation law is introduced at first. In addition, within the scope of second-order accuracy, a brief clarification about the equivalence between governing equations from the integral and differential forms is given in this section as well. Challenges in extending the LG scheme from the node- to cell-centered elements are analyzed in Section 3, and on this basis, correction strategies as well as various corrected schemes are presented subsequently. Truncation errors of the original LG and developed CLG schemes are specifically derived and analyzed in Section 4. Numerical experiments governed by the linear convective, Euler and Navier-Stokes (NS) equations are implemented in Section 5 to give a systematic examination and verification for them. Finally, the paper will be ended with several concluding remarks in Section 6.