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A Surrogate Reduced Order Model of the Unsteady Advection Dominant Problems Based on Combination of Deep Autoencoders-LSTM and POD

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Abstract. Model Order Reduction is an approximation of the main system so that the simplified system retains important features of the main system. Deep learning technology is a recent breakthrough in artificial neural networks that can find more hidden information from the data. In this paper, a non-intrusive reduced order model (NIROM) based on combining deep neural networks (DNNs) and POD abilities, namely FAE-CAE-LSTM is presented. This method combines the obtained features based on Fully connected autoencoders (FAE), Convolutional autoencoders (CAE), and POD and then, a deep Long short-term memory network is trained by obtained features to predict the pressure and velocity fields at future time instances. We investigate the performance of the proposed methodology by solving two well-known canonical cases: a strong shear flow exhibiting the Kelvin–Helmholtz instability, and flow past a cylinder. The performance of the proposed FAE-CAE-LSTM method in future state prediction of the flow is compared with other NIROM methods such as CAE-LSTM, autoencoder-LSTM, autoencoder-DMD and POD-RNN based models. Results show that the FAE-CAE-LSTM method is considerably capable of predicting fluid flow evolution and obtains the best results in the prediction of the pressure and velocity fields in future time instances.

AMS subject classifications: 35Q35, 68T07, 35K40

Key words: Reduced order model, deep learning, autoencoder, long short-term memory, proper orthogonal decomposition.

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1 Introduction

Model order reduction (MOR) is used to reduce the size of the large-scale structure and the dimensions of the dynamic problem. By obtaining the dominant modes, the dimen-

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sions of the corresponding state space are reduced and the approximation of the original model is calculated with acceptable accuracy. Model order reduction is used in various fields, for example, nonlinear large-scale systems [1], ocean modeling [2], sensor position optimization [3], air pollution model [4], shape optimization [5], aerospace [6], optimal control [7] and neutron problems [8]. Model order reduction methods are proposed to achieve the main dynamics of the process through which an efficient approximate computational solution can be obtained [9]. Mathematically, the goal in MOR is to reduce the model, that is, to represent high-dimensional data in a low-dimensional subspace that is necessary to reduce data processing and calculation costs [10].

The first model order reduction works discussed that larger matrices can be simplified to smaller matrices while still having a good approximation [11, 12]. Early MOR methods were published in the 1980s and 1990s. Subsequently, new methods were proposed, such as truncated balanced realization [13], Hankel-norm [14], aggregation method [15], Pade approximation [16], moment matching [17], Routh approximation [18], dynamic mode decomposition (DMD) [19], Koopman analysis [20], and proper orthogonal decomposition (POD), also known as Karhunen-Loeve decomposition and principal component analysis (PCA) [21]. In general, the conventional method of reduced order model is POD because the modes reduced by the POD are mathematically optimal for each dataset. After reducing the model, the next step is to use the reduced modes for modeling. The POD method reduces the number of state variables by mapping the dynamics to a linear subspace provided by the principal components of a data matrix. A very popular technique for this is the Galerkin projection method [22]. The Galerkin method uses spatio-temporal dynamics obtained by model order reduction methods such as POD which can predict them in time rather than governing equations. Galerkin models typically do not take into account spatial variations in flow and become unstable under different conditions, even in conventional cases. Therefore, significant research efforts have been devoted to improving the stability of Galerkin models, and some efforts have focused on formulating without Galerkin. DMD is a method for analyzing the time evolution of a dynamical system and provides an accurate decomposition of complex systems into coherent spatiotemporal structures that may be used for short-time future state prediction and control. It was started in the fluid dynamics community and was first presented by Schmid and Sesterhenn [19]. It is a data-driven equation-free method based on the ability of the singular value decomposition (SVD), and it has been utilized for modal analysis of a diversity of fluid flows.

Recently, deep learning has attracted a lot of attention [23,24]. A deep neural network (DNN) is at the core of deep learning and is a computing system inspired by brain architecture. DNN has shown its strength in overcoming some of the complexities and challenges of machine vision tasks such as large image classification, speech recognition, and more. A deep neural network usually consists of an input layer, an output layer, and several hidden middle layers with several neurons in each layer. In the learning process, each layer converts its input data into an abstract representation and transmits the signal to the next layer. Some activation functions, which act as nonlinear transformers on the